Appendices

This online appendix presents some additional empirical and theoretical results in the paper "Automation and the Rise of Superstar Firms" by Firooz, Liu, and Wang (2024).

Appendix A. Additional tables and figures

Table Appendix A.1. Industries Included in the Sample

ISIC rev4		IFR	
Code	**Label**	**Code**	**Label**
10–12	Manufacture of food products, Manufacture of beverages, Manufacture of tobacco products	10–12	Food products and beverages; Tobacco products
13-15	Manufacture of textiles, Manufacture of wearing apparel, Manufacture of leather and related products	13-15	Textiles, leather, wearing apparel
16, 31	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials, Manufacture of furniture	16	Wood and wood products (incl. furniture)
17-18	Manufacture of paper and paper products, Printing and reproduction of recorded media	17-18	Paper and paper products, publishing & printing
19-22	Manufacture of coke and refined petroleum products, Manufacture of chemicals and chemical products, Manufacture of basic pharmaceutical products and pharmaceutical preparations, Manufacture of rubber and plastics products	19-22	Plastic and chemical products
23	Manufacture of other non-metallic mineral products	23	Glass, ceramics, stone, mineral products n.e.c. (without automotive parts)
24	Manufacture of basic metals	24	Basic metals (iron, steel, aluminum, copper, chrome)
25	Manufacture of fabricated metal products, except machinery and equipment	25	Metal products (without automotive parts), except machinery and equipment
26-27	Manufacture of computer, electronic and optical products, Manufacture of electrical equipment	26-27	Electrical/electronics
28	Manufacture of machinery and equipment n.e.c.	28	Industrial Machinery
29	Manufacture of motor vehicles, trailers and semi-trailers	29	automotive
30	Manufacture of other transport equipment	30	Other transport equipment
D, E	Electricity, gas, steam and air conditioning supply, Water supply; sewerage, waste management, and remediation activities	E	Electricity, gas, water supply

Note: This table shows the corresponding ISIC revision 4 and IFR codes and labels for the industries included in our sample.

Table Appendix A.2. Summary Statistics

	#obs	mean	min	p25	p50	p75	max	s.d.
ln(robot/thousand employees)	117	0.48	-6.57	-1.12	1.02	2.32	5.86	2.36
ln(robots/million hours)	117	0.20	-6.83	-1.34	0.79	1.94	5.30	2.43
top 1% share of sales	117	0.31	0.09	0.22	0.30	0.37	0.77	0.13
top 1% share of employment	104	0.27	0.11	0.21	0.28	0.32	0.46	0.08

Note: This table shows the summary statistics of the data we use in the regressions. The industry-level robot density is measured as the operational stock of industrial robots per thousand employees or per million labor hours. We consider two measures of industry concentration: the sales share and the employment share of the top 1% of firms in the industry. For both measures of concentration, we restrict our sample to industry-year pairs with at least 10 firms.
Source: Authors' calculations using IFR, Compustat, and NBER-CES.

Table Appendix A.3. First-Stage of the IV Regressions for Robot Density and Industry Concentration

Second-stage dependent variable:	top 1% share of sales		top 1% share of emp	
First-stage dependent variable:	$\ln(\frac{\text{robot}}{\text{thousand emp}})$ (1)	$\ln(\frac{\text{robot}}{\text{million hours}})$ (2)	$\ln(\frac{\text{robot}}{\text{thousand emp}})$ (3)	$\ln(\frac{\text{robot}}{\text{million hours}})$ (4)
EURO5 ln(robot/thousand emp)	1.815		1.404	
	(1.214)		(0.936)	
EURO5 ln(robot/million hours)		1.694		1.323
		(1.240)		(0.904)
Observations	117	117	104	104
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
First-stage Effective F-statistic	2.235	1.866	2.251	2.141

Note: This table shows the first-stage results of the IV regression from the empirical specification (2). The second-stage dependent variables are the sales share (first two columns) and employment share (last two columns) of the top 1% of firms. The first-stage dependent variable is the U.S. robot density, measured as the operational stock of industrial robots per thousand workers or million labor hours within the industry. The IV for the U.S. robot density is the one-year lag of the robot density averaged over five European countries (EURO5). The last row shows the first-stage effective F-statistic of Montiel Olea and Pflueger (2013). In all regressions, the industries are weighted by their sales share in the initial year (2007), and the regressions also control for industry and year fixed effects. Standard errors in parentheses are clustered at the industry level. Stars denote the statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table Appendix A.4. Parameter calibration (CES)

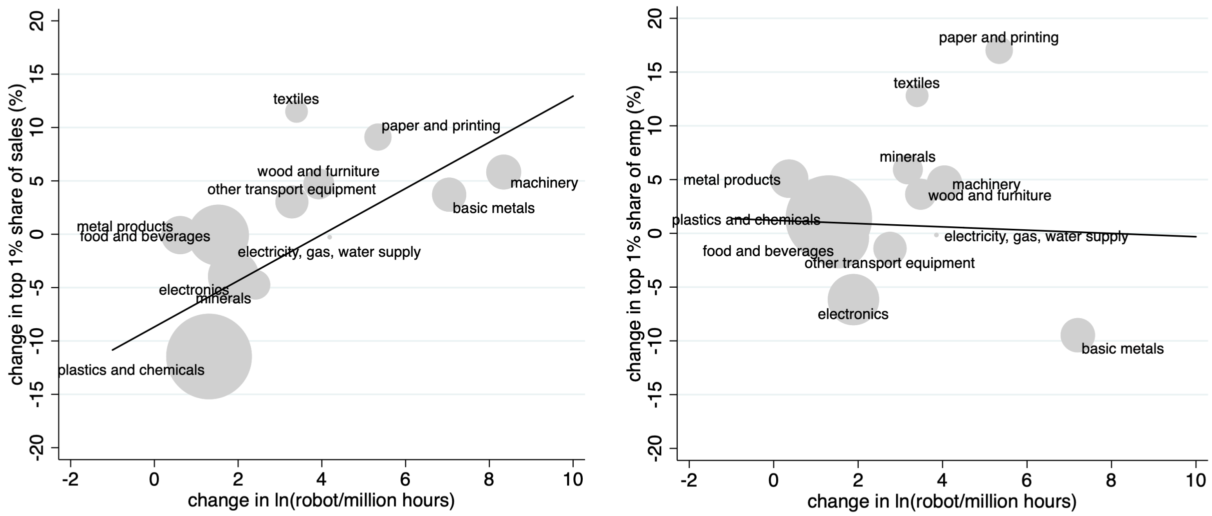
Parameter	Notation	Value	Sources/Matched Moments
Panel A: Parameters calibrated to match external sources			
Discount factor	β	0.99	4% annual interest rate
Inverse Frisch elasticity	ξ	0.5	Rogerson and Wallenius (2009)
Working disutility weight	χ	1	Normalization
Robot depreciation rate	δ_a	0.02	8% annual depreciation rate
Productivity persistence	γ	0.95	Khan and Thomas (2008)
Productivity standard dev.	σ_ϕ	0.1	Bloom et al. (2018)
Demand elasticity parameter	σ	7.39	Matching a markup of 1.156 in the benchmark
Super elasticity	ϵ/σ	0	Imposing constant markups
Panel B: Parameters calibrated to match moments in data			
Relative price of robots	Q_a	44.70	Fraction of automating firms
SD of log automation fixed costs	σ_a	3.12	Employment share of automating firms
Robot input weight	α_a	0.36	Robot density
Elasticity of substitution	η	2.01	Growth rate of robot density

Note: This table shows the calibrated parameters in the counterfactual model with CES aggregation. Panel A reports the externally calibrated parameters and their sources. Panel B shows the parameters calibrated by moment matching.

Figure Appendix A.1. Industry Concentration and Robot Density (per Million Hours)

Panel A. Sales Concentration

Panel B. Employment Concentration

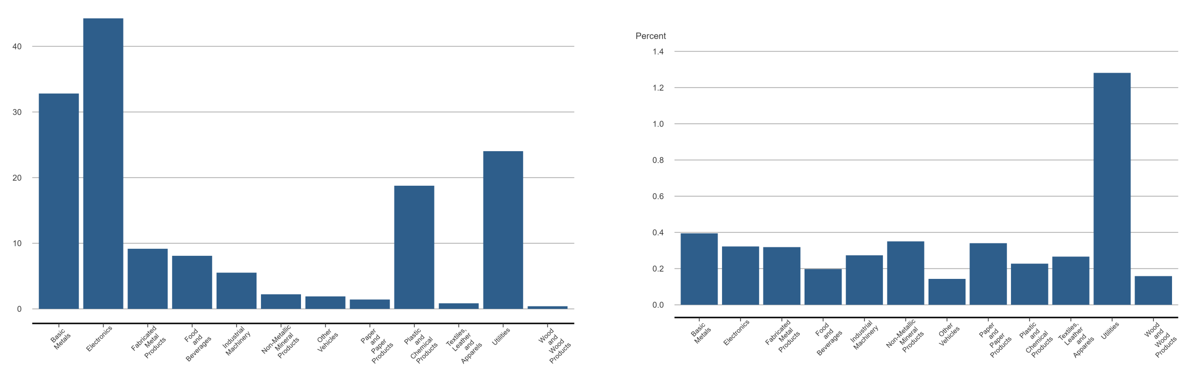


Note: This figure shows the cumulative changes in sales concentration (Panel A) and employment concentration (Panel B) against changes in robot density. The industry concentration is measured by the share of the top 1% of firms within an industry. Robot density is measured by the operational stock of industrial robots per million hours in each industry. The cumulative change is the long difference between the ending value and the starting value of each variable during the years from 2007 to 2018. Since we have an unbalanced panel, we use the first (last) year with non-missing values as the starting (ending) point for calculating the long differences. The circle size indicates an industry's sales share in the initial year (2007). The line shows the prediction from a linear regression weighted by industries' initial sales shares. The slope coefficient for sales concentration (Panel A) is 0.022 with a standard error of 0.008. The slope coefficient for employment concentration (Panel B) is −0.0015 with a standard error of 0.010.
Source: IFR, NBER-CES, Compustat, and authors' calculation.

Figure Appendix A.2. Distribution of robot density and capital equipment intensity across industries

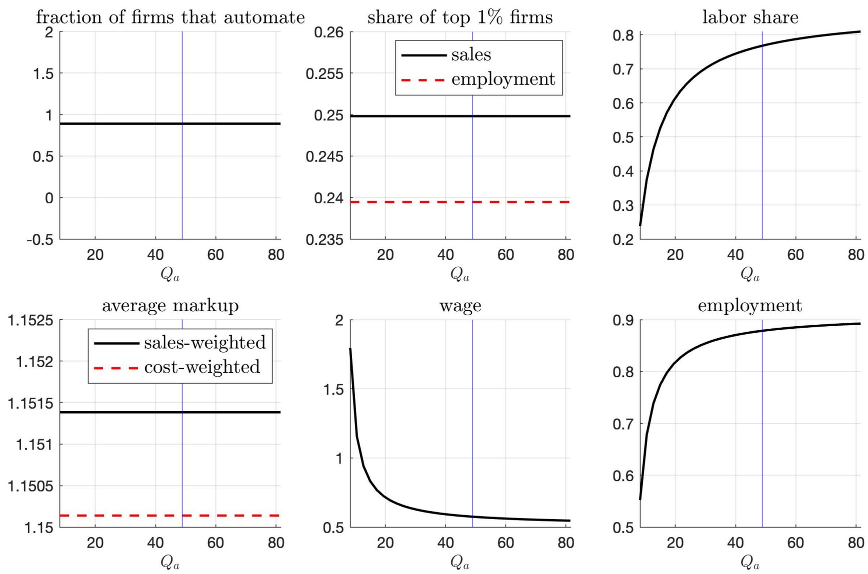
Panel A. Robot density

Panel B. Capital equipment intensity



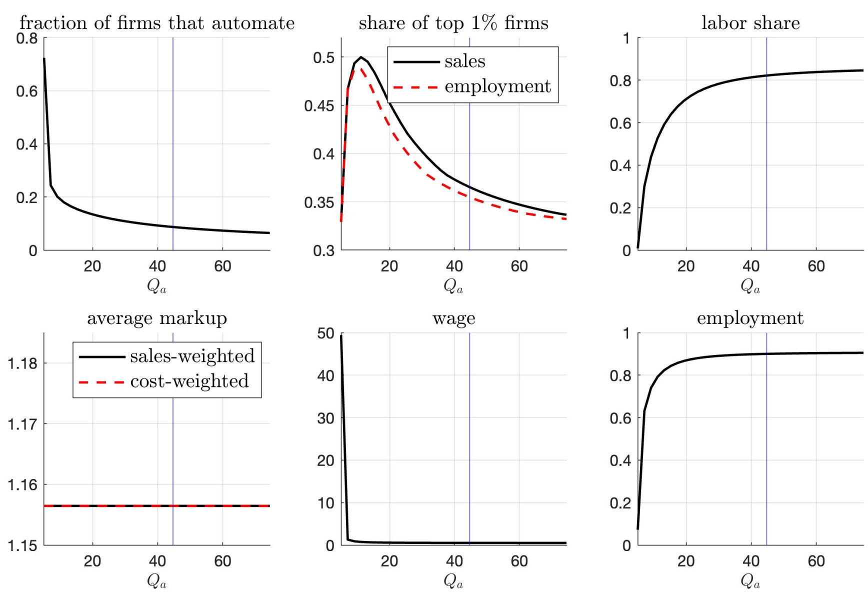
Note: This figure shows the distribution of robot density (Panel A) and of capital equipment intensity (Panel B) in the year 2018 across the 12 two-digit manufacturing industries in our sample. Robot density is measured by the operational stock of industrial robots per thousand workers in an industry. Capital equipment intensity is measured by the ratio of the nominal value of capital equipment to the nominal value added in an industry.
Source: IFR, NBER-CES, Bureau of Economic Analysis, and authors' calculation.

Figure Appendix A.3. Aggregate Variables (no fixed cost of automation)



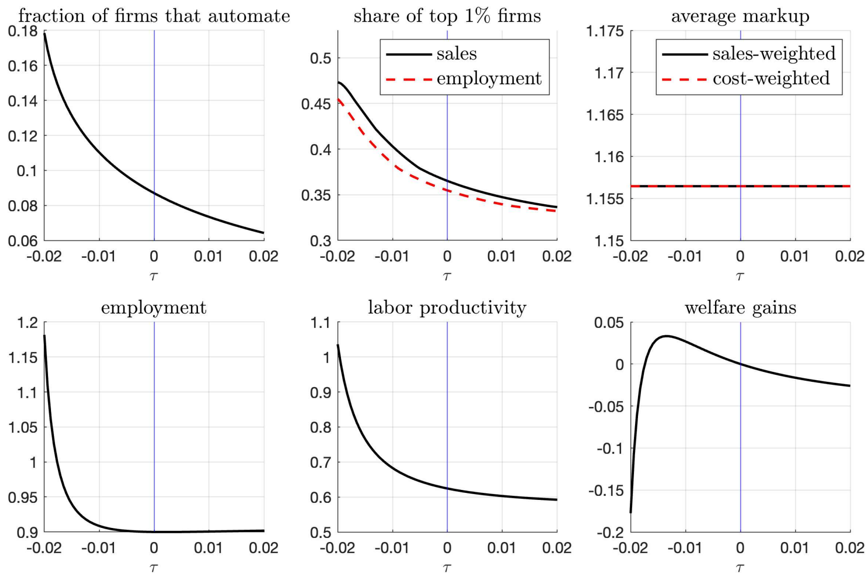
Note: This figure shows the steady-state effects of changes in the robot price Q_a on the fraction of firms that automate, the share of the top 1% of firms, the labor share, the average markup, the wage rate, and employment in the counterfactual model with no fixed cost of automation. The vertical blue line indicates the calibrated value of robot price Q_a.

Figure Appendix A.4. Aggregate Variables (CES)



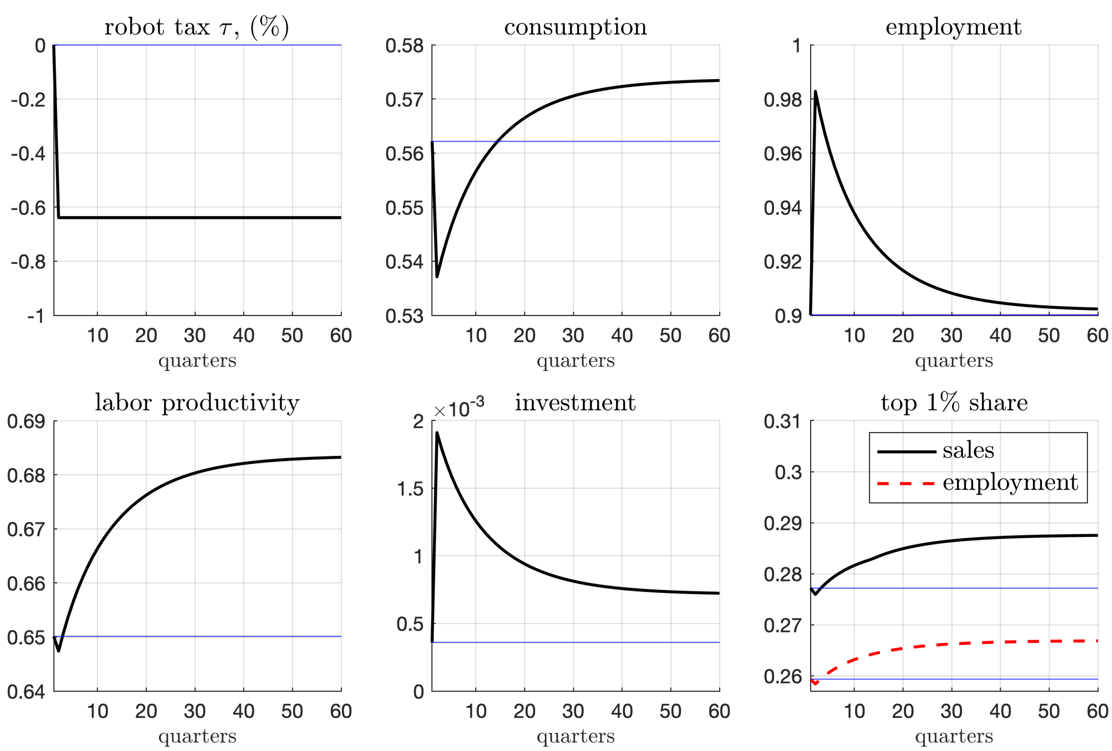
Note: This figure shows the effects of changes in the robot price Q_a on the fraction of firms that automate, the share of the top 1% of firms, the labor share, the average markup, the wage rate, and employment in the counterfactual model with a CES demand system. The vertical blue line indicates the calibrated value of robot price Q_a.

Figure Appendix A.5. Steady-State Effects of Taxing Robots (CES)



Note: This figure shows the effects of imposing a tax τ on the stock value of robots on aggregate variables and welfare in the steady state in the model with a CES demand system. Welfare gains are measured by the consumption equivalent (percent) relative to the laissez-faire economy with $\tau = 0$.

Figure Appendix A.6. Transition Paths Under the Optimal Robot Tax



Note: This figure shows the dynamic effects of imposing a permanent robot subsidy of 0.64%, corresponding to the optimal subsidy rate in the dynamic model under the benchmark calibration.

Appendix B. Derivations

To simplify the intermediate producers' problem in equation (19), rewire the value function so that s is not a state variable:

$$\begin{aligned}
V_t(\phi_t; s_t) = & \max_{p_t, y_t, N_t, A_t} \Big[p_t(\phi_t) y_t(\phi_t) - W_t N_t(\phi_t) - r_{a,t} A_t(\phi_t) - s_t \phi_t \mathbb{1}\{A_t(\phi_t) > 0\} \\
& + \rho_{t,t+1} E_{\phi_{t+1}|\phi_t} \int_{s_{t+1}} V_{t+1}(\phi_{t+1}; s_{t+1}) dF(s_{t+1}) \Big] \\
& = \max \Bigg\{ \underbrace{\max_{p_t, y_t, N_t, A_t} \Big[p_t(\phi_t) y_t(\phi_t) - W_t N_t(\phi_t) - r_{a,t} A_t(\phi_t) + \rho_{t,t+1} E_{\phi_{t+1}|\phi_t} \int_{s_{t+1}} V_{t+1}(\phi_{t+1}; s_{t+1}) dF(s_{t+1}) \Big]}_{\equiv V_t^a(\phi_t)} \\
& - s_t \phi_t, \underbrace{\max_{p_t, y_t, N_t} \Big[p_t(\phi_t) y_t(\phi_t) - W_t N_t(\phi_t) + \rho_{t,t+1} E_{\phi_{t+1}|\phi_t} \int_{s_{t+1}} V_{t+1}(\phi_{t+1}; s_{t+1}) dF(s_{t+1}) \Big]}_{\equiv V_t^n(\phi_t)} \Bigg\} \\
& = \max\{V_t^a(\phi_t) - s_t \phi_t, V_t^n(\phi_t)\}
\end{aligned} \tag{B.1}$$

The firm with productivity ϕ_t chooses $A_t(\phi_t) > 0$ if and only if $s_t \leq s_t^*(\phi_t) \equiv \frac{V_t^a(\phi_t) - V_t^n(\phi_t)}{\phi_t}$.

The value of an automating firm can be written as

$$V_t^a(\phi_t) = \max_{p_t, y_t, N_t, A_t > 0} \Big[p_t(\phi_t) y_t(\phi_t) - W_t N_t(\phi_t) - r_{a,t} A_t(\phi_t) \Big] + \rho_{t,t+1} E_{\phi_{t+1}|\phi_t} \int_{s_{t+1}} V_{t+1}(\phi_{t+1}; s_{t+1}) dF(s_{t+1}) \tag{B.2}$$

The value of a non-automating firm can be written as

$$V_t^n(\phi_t) = \max_{p_t, y_t, N_t} \Big[p_t(\phi_t) y_t(\phi_t) - W_t N_t(\phi_t) \Big] + \rho_{t,t+1} E_{\phi_{t+1}|\phi_t} \int_{s_{t+1}} V_{t+1}(\phi_{t+1}; s_{t+1}) dF(s_{t+1}) \tag{B.3}$$

To compute the automation cutoff $s_t^*(\phi_t)$, we can write:

$$s_t^*(\phi_t)\phi_t = V_t^a(\phi_t) - V_t^n(\phi_t) \tag{B.4}$$

$$= \max_{p_t, y_t, N_t, A_t} \Big[p_t(\phi_t) y_t(\phi_t) - W_t N_t(\phi_t) - r_{a,t} A_t(\phi_t) \Big] - \max_{p_t, y_t, N_t} \Big[p_t(\phi_t) y_t(\phi_t) - W_t N_t(\phi_t) \Big], \tag{B.5}$$

which gives Eq. 30 in the text.

Appendix C. Solution Algorithm

Appendix C.1. Steady state

In the steady state, the rental rate of robots is

$$r_a = Q_a(\frac{1}{\beta} - 1 + \delta_a). \tag{C.1}$$

There are three loops to solve for the steady state. The Y loop is outside of the W loop and the W loop is outside of the q loop.

Y loop: Use bisection to determine the aggregate final goods and other aggregate variables.

1. Guess aggregate final goods Y.
2. Compute W and firms' relative production $q(\phi)$ in the W loop as explained below.
3. Given the equilibrium wage rate, compute other aggregate variables by finding Y using the bisection method:
   (a) Given the solved relative production $q(\phi)$, we have $y(\phi) = q(\phi)Y$.
   (b) Given the robot price Q_a and the wage rate W, compute the marginal costs $\lambda^n(\phi)$ and $\lambda^a(\phi)$ by eq. (25) and (27), and we can get $A(\phi)$ and $N(\phi)$ from eq. (23), (24), and (26).
   (c) The aggregate employment and robot stock are determined by eq. (34) and eq. (35).
   (d) Consumption C is determined by eq. (7).
   (e) The steady state aggregate investment in robots I_a is from (36).
   (f) Compute Y^{new} using the resource constraint (33). Stop if Y converges.
      i. If $Y = Y^{\text{new}}$, Y and all other aggregate variables are found.
      ii. If $Y > Y^{\text{new}}$, reduce Y. Go back to Step 1.
      iii. If $Y < Y^{\text{new}}$, increase Y. Go back to 1.

W loop: Use bisection to determine the wage rate.

1. Guess a wage W.
2. Compute firms' relative production $q(\phi)$ in the q loop as explained below.
3. Check whether the Kimball aggregator (9) holds.
   (a) If LHS = RHS, the wage rate is found and jump out of W loop to Y loop.
   (b) If LHS > RHS, increase W to reduce $q(\phi)$ according to eq. (10). Go back to Step 2.
   (c) If LHS < RHS, reduce W to raise $q(\phi)$ according to eq. (10). Go back to Step 2.

q loop: Find the relative production.

1. Given the prices Q_a and W, the marginal cost of production is determined by eq. (25) for the automation technology and by eq. (27) for the labor-only technology.
2. Guess a demand shifter D.
3. Use eq. (10) to solve for the relative output $q(\phi)$ for each ϕ, for firms with and without robots.

(a) The right-hand side of (10) is a function of $q(\phi)$ by plugging in (14).
(b) The price in the left-hand side is the marginal cost in (25) or (27) times the markup in (16), which is also a function of $q(\phi)$.
(c) Use the bisection method to solve for $q(\phi)$ in eq. (10).

4. Compute the automation decisions.
   (a) Compute $y(\phi) = q(\phi)Y$ with and without robots.
   (b) Compute the demand for $A(\phi)$ and $N(\phi)$ with and without robots from eq. (23), (24), and (26).
   (c) For each productivity ϕ, compute the profits with and without robots and thus get the automation cutoffs $s^*(\phi)$ according to (30), and thus the automation probability $F(s^*(\phi))$.
5. Given the automation decisions, compute D^{new} by (11). Stop if D converges. Otherwise, go back to Step 2 and repeat until D converges.
   (a) If $D = D^{\text{new}}$, D and $q(\phi)$ are found and jump out of q loop to W loop.
   (b) If $D > D^{\text{new}}$, reduce D. Go back to Step 2.
   (c) If $D < D^{\text{new}}$, increase D. Go back to Step 2.

Appendix C.2. Transitional dynamics

We assume that the economy is in the steady state at $t = 1$ and Q_a unexpectedly decreases by 40% in period 2 and remains deterministically constant afterward.

Given an exogenous path of $\{Q_{a,t}\}_{t=1}^{T}$, we solve the economy's transition path as follows:

1. Ensure that T is sufficiently large so that the economy reaches its new steady state by time T. For example, set $T = 300$. The economy begins at its initial steady state at $t = 1$ and reaches the new steady state at $t = T$, an unexpected change in robot prices.
2. Make initial guesses for the sequence of stochastic discount factors (SDFs), the sequence of aggregate output, and $r_{a,2}$. Set $\{\rho_{t,t+1}^{(init)}\}_{t=2}^{T-1} = \beta$, $\{Y_t^{(init)}\}_{t=2}^{T-1} = Y_T$, and $r_{a,2}^{(init)}$ in between $r_{a,1}$ and $r_{a,T}$.[1]
3. For each $t = 2, 3, ..., T-1$, given $Q_{a,t-1}$, $Q_{a,t}$, $r_{a,2}^{(init)}$, $\rho_{t-1,t}^{(init)}$, and $Y_t^{(init)}$, solve for the equilibrium as follows:
   (a) The rental rate of robots for $t = 3, 4, ..., T-1$ is given by

$$Q_{a,t} = \rho_{t,t+1}^{(init)} \left[r_{a,t+1} + Q_{a,t+1}(1-\delta_a) \right].$$

$$\Rightarrow r_{a,t+1} = Q_{a,t}/\rho_{t,t+1}^{(init)} - Q_{a,t+1}(1-\delta_a).$$

$$\Rightarrow r_{a,t} = Q_{a,t-1}/\rho_{t-1,t}^{(init)} - Q_{a,t}(1-\delta_a). \tag{C.2}$$

[1]Since aggregate investment depends on the next period's aggregate robot stock, another variable besides the SDF needs to be guessed. An alternative approach is to guess the sequence of robot stocks $\{A_t^{(init)}\}_{t=2}^{T-1}$ and solve for Y, W, and q loops, as described in the steady state solution algorithm. In practice, this approach is slower and does not improve convergence.

(b) Given $r_{a,t}$ and $Y_t^{(init)}$, solve the W and q loops outlined in the steady state solution algorithm. This yields W and firms' relative production $q_(\phi_t)$.

(c) Similar to the Y loop in the steady state solution algorithm, compute other variables as follows:

i. Given $r_{a,t}$ and W, compute the marginal costs $\lambda_t^n(\phi_t)$ and $\lambda_t^a(\phi_t)$ given by eq. (25) and (27), and solve for $A_t(\phi_t)$ and $N_t(\phi_t)$ from eq. (23), (24), and (26).

ii. The aggregate employment and robot stock are determined by eq. (34) and eq. (35).

iii. Aggregate consumption C_t is determined by eq. (7).

(d) Compute aggregate investment in robots, $I_{a,t} = A_{t+1} - (1 - \delta_a)A_t$.

(e) Compute aggregate output for each t using the resource constraint (33):

$$Y_t^{(\text{new})} = C_t + Q_{a,t} I_{a,t} + \int_{\phi_t} \int_0^{s_t^*(\phi_t)} s_t \phi_t \, dF(s_t) \, dG(\phi_t). \tag{C.3}$$

(f) Compute stochastic discount factors for each t: $\rho_{t,t+1}^{(\text{new})} = \beta \frac{C_t}{C_{t+1}}$.

(g) Update $r_{a,2}^{(\text{new})} = A_2 - A_1$. Notice that $r_{a,2}$ is not given by equation (C.2) because the shock at period 2 is unexpected. Instead, $r_{a,2}$ is determined such that robot demand equals the pre-determined robot supply at period 1, i.e., $A_2 = A_1$.

4. Continue iterating until the sequences of SDFs, aggregate output, and $r_{a,2}$ converge, i.e., $\text{dist}(\{\rho_{t,t+1}^{(\text{new})}\}_{t=1}^T, \{\rho_{t,t+1}^{(init)}\}_{t=1}^T) < 10^{-6}$, $\text{dist}(\{Y_t^{(\text{new})}\}_{t=1}^T, \{Y_t^{(init)}\}_{t=1}^T) < 10^{-6}$, and $|r_{a,2}^{(\text{new})} - r_{a,2}^{(\text{init})}| < 10^{-6}$. Here, the distance function is defined as $\text{dist}(f^{(\text{new})}, f^{(init)}) = \frac{\left(\sum_t (f^{(\text{new})}(t) - f^{(init)}(t))^2\right)^{1/2}}{1 + \left(\sum_t f^{(init)}(t)^2\right)^{1/2}}$, as in Judd (1998). If any of them does not converge, update our initial guess and start again from Step 3:

$$\rho_{t,t+1}^{(init)} = \eta \rho_{t,t+1}^{(init)} + (1 - \eta) \rho_{t,t+1}^{(\text{new})},$$

$$Y_t^{(init)} = \eta Y_t^{(init)} + (1 - \eta) Y_t^{(\text{new})},$$

$$r_{a,2}^{(init)} = \eta r_{a,2}^{(init)} + (1 - \eta) r_{a,2}^{(\text{new})},$$

with $\eta = 0.99$.

Appendix D. Calibrating the mean fixed cost of automation

In our benchmark calibration, we assume that the log fixed costs of automation have a mean of zero because we do not have an additional data moment in the manufacturing sector to calibrate this parameter. To examine the robustness of our results, we now calibrate the mean of the log-normal distribution of the fixed costs (denoted by μ_a) by targeting a data moment in the whole economy. The moment that we target is the ratio of the robot use rate among firms between the 50th and 75th percentile of the employment distribution (1.7%) to the average robot use rate among all firms in the whole economy

(2%), taken from the 2019 ABS documented by Acemoglu et al. (2022). This moment ($\frac{1.7}{2} = 0.85$) also captures the skewness of using robotics across U.S. firms.[2]

Table Appendix D.1 presents the calibrated parameters. Table Appendix D.2 shows that the calibrated model exactly matches all the five moments in the data. In this calibrated model, as shown in Figure Appendix D.1, the predicted steady-state relations between the robot price and the macroeconomic variables are qualitatively similar to those in the benchmark model. Quantitatively, a 40% decline in the robot price raises the sales share of the top 1% of firms by about 1.23 percentage points (from 26% to 27.23%) and the employment share of the top 1% of firms by about 0.9 percentage points. Therefore, this model predicts that the decline in the robot price explains about 41% of the observed increases in sales concentration (1.23 out of 3 percentage points) and about 18% of the divergence between sales and employment concentration (0.32 out of the 1.8 percentage points). These magnitudes of the contributions from automation to industry concentration are slightly smaller than, but comparable to, those in the benchmark model. Thus, our main results are robust to calibrating the mean fixed cost of automation.

Table Appendix D.1. Parameters (calibrating the mean fixed cost of automation)

Parameter	**Notation**	**Value**	**Matched Moments**
Relative price of robots	Q_a	46.47	Fraction of automating firms
Mean of log automation fixed costs	μ_a	-0.32	Skewness of robot use rate
SD of log automation fixed costs	σ_a	3.09	Employment share of automating firms
Robot input weight	α_a	0.34	Robot density
Elasticity of substitution	η	2.03	Growth rate of robot density

Note: This table shows the calibrated parameters by moment matching. Compared to the benchmark model, we calibrate an additional parameter, which is the mean of the log-normal distribution of the fixed cost of automation (μ_a) by matching the skewness of robot use rate measured by the ratio of the robot use rate among firms between the 50th and the 75th percentile of the employment distribution to the average robot use rate among all firms in the whole economy in the ABS data documented by Acemoglu et al. (2022).

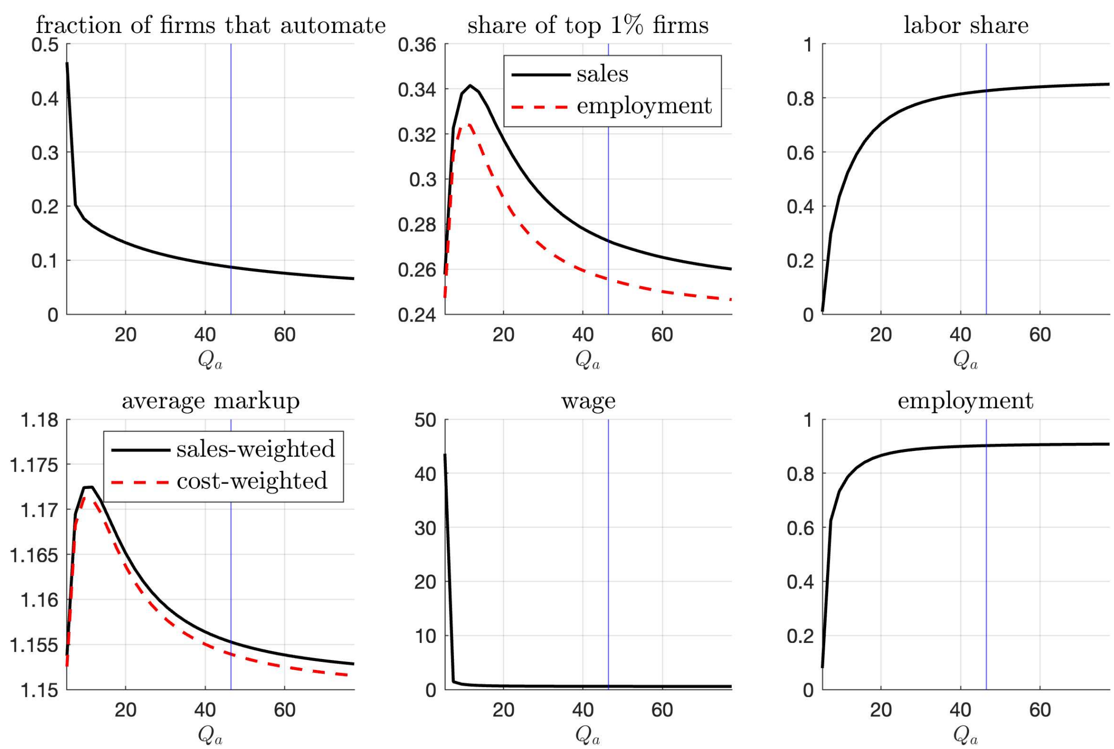
[2]This moment is only available for the whole economy and not for the manufacturing sector. Hence, we report these results here as a robustness check.

Table Appendix D.2. Matched Moments (calibrating the mean fixed cost of automation)

Moments	**Data**	**Model**
Fraction of automating firms	8.7%	8.7%
Skewness of robot use rate	0.85	0.85
Employment share of automating firms	45.1%	45.1%
Robot density	0.02	0.02
Growth rate of robot density	300%	300%

Note: This table shows the targeted data moments and the simulated moments by the model. The first three data moments are based on the ABS data (taken from Acemoglu et al., 2022), and the last two moments are authors' calculations using IFR and NBER-CES data. The skewness of robot use rate is measured by the ratio of the robot use rate among firms between the 50th and the 75th percentile of the employment distribution to the average robot use rate among all firms in the whole economy in the ABS data documented by Acemoglu et al. (2022).

Figure Appendix D.1. Aggregate Variables (calibrated mean fixed cost of automation)



Note: This figure shows the effects of changes in the robot price Q_a on the fraction of firms that automate, the share of the top 1% of firms, the labor share, the average markup, the wage rate, and employment in the model with a calibrated value of the mean fixed cost of automation. The vertical blue line indicates the calibrated value of robot price Q_a.