# Optimal Intermediary Contracts

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**Abstract**

We study the role of pledgeability in shaping optimal intermediary contracts. We develop a model in which financial intermediaries offer deposit contracts to partially insure lenders against idiosyncratic risks and extend collateralized loans to borrowers with limited commitment. The model shows that a nonmonotonic relationship between contract terms and pledgeability emerges in general equilibrium. In a stochastic version of the model, we investigate how pledgeability affects the bank's choice of contracts and the endogeneity of bank runs.

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**Keywords**: optimal deposit contract, optimal loan contract, pledgeability, bank risk, Bankruptcy Abuse Prevention and Consumer Protection Act.

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# 1 Introduction

Financial intermediaries accept deposits and allocate these funds between loans and securities, contracting simultaneously with depositors and borrowers. The terms of these contracts therefore reflect the composition of intermediary assets, the risks associated with lending, and the degree of risk sharing among depositors. How do frictions in the loan market shape these outcomes? In this paper, we study how such frictions affect intermediary portfolios, loan and deposit contracts, financial stability, and the endogenous occurrence of banking crises.

We develop a general-equilibrium framework that jointly models the deposit and loan markets. Our framework extends the canonical Diamond-Dybvig model along two dimensions.[1] First, following Donaldson, Piacentino, and Thakor (2018), we introduce a limited-commitment problem in the loan market. Donaldson et al. enforce repayment by excluding defaulting borrowers from deposit services. We instead adopt the more traditional approach of allowing borrowers to pledge assets as collateral, as in Kiyotaki and Moore (1991).[2] The severity of the financial friction is represented by asset pledgeability: the fraction of a borrower's collateral that the bank can recover following default. We then examine how pledgeability affects equilibrium contracts in both the loan and deposit markets.

Second, we expand the bank's portfolio choice by introducing a safe, low-return, long-term asset, thereby allowing for a portfolio-diversification channel. Although loans offer a higher return than the safe asset, lower pledgeability tightens the borrower's repayment constraint, eventually forcing banks to allocate some resources to the safe asset.

[1]Our work builds on a long tradition of models analyzing the role of banks and secondary markets in providing partial insurance, including Bryant (1980), Jacklin (1987), Bhattacharya and Gale (1987), Hellwig (1994), Diamond (1997), Holmstrom and Tirole (1998), Von Thadden (1999), Allen and Gale (2003), Caballero and Krishnamurthy (2004), and Farhi et al. (2009).

[2]Our model also builds on a rich literature analyzing optimal loan contracts under information frictions. For discussions of collateral in loan contracts, see, for example, Williamson (1987), Besanko and Thakor (1987), Bernanke and Gertler (1990), Berger and Udell (1990), and Dowd (1992).

Our first contribution is to fully characterize how pledgeability affects equilibrium contract terms in both markets. Although this exercise is natural, to our knowledge it has not previously been carried out in a unified general-equilibrium setting. We show that the competitive equilibrium falls into one of three regions, depending on the pledgeability. In the high-pledgeability region, the borrower's repayment constraint does not bind. In the medium-pledgeability region, the repayment constraint binds, but the bank does not invest in the low-return safe asset. In the low-pledgeability region, the repayment constraint binds and the bank holds some safe asset.

A central and counterintuitive result is that several contract terms, including collateral and loan size, can vary nonmonotonically with pledgeability, whereas the loan rate is weakly increasing in pledgeability. In particular, an improvement in pledgeability can *increase* the amount of collateral, *reduce* the size of the loan, and leave borrowers worse off.[3] On the deposit side, the long-term rate increases with pledgeability, whereas the response of the short-term rate depends on depositors' elasticity of intertemporal substitution (EIS).[4]

The non-monotonicity result is driven by the elasticity of supply and demand. While higher pledgeability relaxes the repayment constraint and stimulates loan demand, competition for loans bids up loan rates. This change in rate has a profound impact on loan size, and the results depend on the slope of the supply curve, which in turn depends on the depositor's EIS. Specifically, loan size falls with pledgeability when depositors' EIS is less than one and rises when it is greater than one. Given the higher return on the loan market, how to allocate the resources among the depositors again depends on the depositor's EIS. As is well known in the Diamond-Dybvig framework, the inverse of the EIS–that is, the CRRA–is important for pinning down the relationship between patient and impatient consumption. Under the conventional Diamond-Dybvig assumption

[3]Kaplan and Zingales (1997) derive a nonmonotonicity condition in an economy with a financial-market-tightness parameter. Because market tightness enters the borrower's cost function as a parameter, their condition depends on changes in the curvature of the production function relative to changes in the curvature of the cost function.

[4]In the Appendix, we show that the short-term deposit rate can be nonmonotone if the borrowers have some market power in the loan market.

of the depositor's EIS being less than 1, both short-term and long-term deposit rates increase with pledgeability. When the EIS exceeds one, however, the short-term rate falls while the long-term rate rises. For a similar reason, the response of collateral depends on borrowers' EIS.

Our analysis complements Kaplan and Zingales (1997), who model financing costs as a *reduced-form* function of financial constraints. In their framework, the financing cost is represented by the wedge between a firm's internal and external costs of funds, and they derive conditions under which loan-contract terms vary nonmonotonically with financial constraints. We instead model the underlying friction directly as a loan-repayment constraint and *endogenize* the interest-rate differential in general equilibrium. Consequently, the wedge between internal and external financing costs may itself vary nonmonotonically with the severity of the friction. Our results, therefore, show that this interest-rate differential need not be a sufficient statistic for the tightness of a firm's financial constraint.[5]

Our second contribution concerns financial stability. We characterize how pledgeability affects the optimal demand-deposit contract and the endogenous occurrence of bank runs. For this purpose, we extend the baseline model to allow for stochastic loan returns and default. In the presence of default risk, the bank may invest enough in safe technology to offer a run-proof contract. Under such a contract, even when borrowers default, the bank retains sufficient resources to induce depositors to wait until its assets mature rather than liquidate them prematurely. Alternatively, the bank may choose a riskier portfolio and offer a run-admitting contract. If default occurs, the bank lacks sufficient resources to induce depositors to wait, and a bank run follows.

Pledgeability affects this choice through the equilibrium loan rate. Low pledgeability suppresses loan demand and depresses the loan rate. Because loans offer a relatively low return while remaining exposed to default, the bank invests more heavily in the

[5]In Kaplan and Zingales (1997), "firm is considered more financially constrained as the wedge between its internal and external cost of funds increases." They are "agnostic on whether the wedge between the cost of internal and external funds is caused by hidden information problems,... or agency problems..."

safe asset. This portfolio smooths depositor consumption across both time and states, providing relatively high payments even when loans default. A run-proof contract is therefore optimal when pledgeability is low. By contrast, high pledgeability raises the loan rate and encourages the bank to invest more heavily in risky loans. The resulting portfolio admits bank runs in the event of default, but it also permits higher depositor payments in normal times. When these gains outweigh the losses incurred during crises, the run-admitting contract is optimal.

The model thus yields a seemingly paradoxical conclusion: alleviating financial frictions in the loan market can make banks more fragile. The underlying mechanism is straightforward. Greater pledgeability improves the efficiency and profitability of lending in normal times, inducing banks to accept greater exposure to crisis risk. Banking crises are therefore tolerated in equilibrium precisely because the loan market performs better outside crisis states.

## Literature Review

Our paper relates to several strands of literature. Following Diamond and Dybvig (1983), a vast body of work explores the role of banks in providing partial insurance to consumers facing idiosyncratic liquidity shocks (see footnote 1). Similarly, a substantial literature examines optimal loan contracts and credit rationing of different forms (e.g., Kiyotaki and Moore, 1997; Holmes and Tirole, 1998; Jermann and Quadrini, 2012, Bai et al., 2026, among others). We bridge these two strands of research by studying an economy where deposit contracts and loan contracts are determined simultaneously in general equilibrium, allowing financial frictions on the lending side, specifically limited commitment by borrowers, to feed back into the contract terms.

There is a substantial body of literature modeling the limited commitment problem in the banking setup. Notably, Donaldson, Piacentino and Thakor (2018) consider Our paper is related to several strands of literature. Following Diamond and Dybvig (1983), a vast body of work explores the role of banks in providing partial insurance to consumers facing idiosyncratic liquidity shocks (see footnote 1). Similarly, a substantial literature examines optimal loan contracts and credit rationing of different forms (e.g., Kiyotaki and Moore, (1997); Holmes and Tirole, (1998); Jermann and Quadrini, (2012), Bai et al., (2026), among others). Our paper bridges these two strands of research by studying an economy where deposit contracts and loan contracts are determined simultaneously in general equilibrium, allowing financial frictions on the lending side, specifically limited commitment by borrowers, to feed back into the contract terms.

There is a substantial body of literature modeling the limited commitment problem in the banking setup. Notably, Donaldson, Piacentino, and Thakor (2018) consider an economy in which borrowers cannot commit to repaying loans. With safeguarding services provided, the bank partially insures borrowers. The bank enforces repayment by threatening to exclude defaulting borrowers from valuable deposit services. In Antinolfi and Prasad (2008), collateral serves as a means of liquidity provision. By pooling collateral, banks allocate resources more efficiently than individuals. The debt repayment constraint may not bind for the bank, even though it would bind for individuals. Amador and Bianchi (2024) models banks as borrowers that acquire liquidity through bond issuance. Their framework includes a stochastic-return capital asset as an outside option in the bank's asset structure. Their work focuses on the bank's default decision, while ours examines the relationship between pledgeability and intermediary contracts. Regarding the literature on limited commitment and financing choice, our paper is closely related to Rocheteau et al.(2018). They study the transmission of nominal interest rates to real lending rates, focusing on the firm's financing choices, whereas we focus on the bank's portfolio choice.

There is a large empirical literature examining collateral in the optimal loan contract. Along the extensive margin, Hester (1972), Berger and Udell (1990,1995), and Klapper (1999) provide empirical support for the hypothesis that collateral is used by less creditworthy borrowers. John et al. (2003) present evidence that there is a positive, extensive relationship between collateral and loan rates. They explain how lenders price agency risk into the loan rate. Along the intensive margin, Benmelech and Bergman (2009) argue that the empirical work suffers from selection bias inherent in collateral's extensive margin. In their paper, collateral redeployability serves as a proxy for the intensity of collateral pledged. They present evidence that there is a negative relationship between collateral values on both loan rates and loan size.[6] The nonmonotonic contract terms in our paper provide a potential explanation that reconciles the conflicting findings across studies of the extensive and intensive margins.

The rest of the paper is organized as follows. Section 2 sets up the model environment. Section 3 derives the theoretical results. Section 4 extends the model to allow for stochastic loan returns and to solve the optimal demand deposit contract. Section 5 concludes.

# 2 The Model

## 2.1 Environment

The model environment is based on Diamond and Dybvig (1983) with the introduction of an additional type of agent: borrowers. There are three time periods indexed by $t = 0, 1, 2$. There are two types of agents in this economy: depositors and borrowers. The measure of depositors is normalized to 1 while the measure of borrowers is $n$.

Each depositor is endowed with one unit of capital in $t = 0$ and nothing in periods $t = 1, 2$. Depositors have access to a long-term investment technology that turns 1 unit of capital at $t = 0$ into $\underline{R} > 1$ units of consumption good at $t = 2$ or 1 unit of consumption good at $t = 1$. There is also a storage technology that can transform capital into consumption good at a one-for-one rate in either period.

Depositors are identical in period 0. At date $t = 1$, each depositor receives an idiosyncratic preference shock. With probability $\lambda$, the depositor is impatient, meaning

[6] With both extensive and intensive margins, biased estimates are present if riskier firms are required to pledge more collateral. Because the quantity of collateral may be correlated with unobserved characteristics that affect loan rates, studies finding a positive relationship between loan rates and the presence of collateral are tainted by selection bias.

he derives utility exclusively from consuming in $t = 1$. With probability $1 - \lambda$, the depositor is patient, meaning he values consumption in $t = 2$.

Let $u(x_t)$ denote the utility function of the depositor, where $x_t$ is the consumption in period $t$. The utility function is strictly increasing, strictly concave, and normalized to $u(0) = 0$. Whether a depositor is patient or impatient is his private information. By the law of large numbers, $\lambda$ is also the fraction of depositors in the population who are impatient.

Borrowers have access to a technology that turns 1 unit of capital at $t = 0$ into $\bar{R} > \underline{R}$ units of consumption good at $t = 2$ but nothing at $t = 1$. They are not endowed with capital. However, they can produce capital by incurring a utility cost at $t = 0$. Let $c(k)$ be the cost function of producing $k$ units of capital at $t = 0$, where $c', c'' > 0$ and $c'(0) = 0$. Borrowers consume at $t = 2$ with utility function $v(x)$, where $v' > 0 > v''$.

Following Diamond and Dybvig (1983), depositors have an incentive to form a coalition that operates as a bank, offering its members a deposit contract that provides insurance against idiosyncratic consumption shocks. Borrowers have access to a more productive technology. Absent additional frictions, the bank therefore lends its funds to borrowers at ($t = 0$) in exchange for a higher return.

Borrowers, however, cannot commit to repay. After receiving a loan, a borrower may renege by diverting the loan and the borrower's own capital, transforming these assets into consumption goods, and leaving only the pledgeable portion of the collateral to the bank. To capture the loss of productivity associated with diversion, we assume that diverted assets yield a return of 1, rather than $\bar{R}$. The borrower's total collateral consists of the loan plus the borrower's own capital.[7] Let $\chi$ denote the fraction of this collateral that the bank can recover following diversion. The borrower consumes the remaining fraction, 1-$\chi$.[8]

[7] Boot et al. (1991) explain the use of collateral as a response to borrowers' hidden actions and private information.

[8] Related diversion technologies are modeled as cash-flow diversion in Biais et al. (2007) and DeMarzo and Fishman (2007).

## 2.2 No Loan Market

We first calculate the payoffs of banks and borrowers without the loan market. In this setup, the borrower side's friction does not affect the deposit market at all. As a coalition of depositors, the bank seeks to maximize the expected welfare of its depositors. Formally, the deposit contract solves

$$\hat{W}_D = \max_{x_1, x_2} \left[\lambda u\left(x_1\right) + \left(1-\lambda\right) u\left(x_2\right)\right] \tag{1}$$

$$\text{s.t. } \left(1-\lambda x_1\right) \underline{R} = \left(1-\lambda\right) x_2 \tag{2}$$

$$x_2 \geq x_1 \tag{3}$$

where (2) is the resource constraint and (3) is the incentive constraint for late depositors to wait until $t = 2$ to withdraw. The bank liquidates $\lambda x_1$ from its investment in $t = 1$ to pay impatient depositors and leave the rest until $t = 2$ with return $\underline{R}$ to pay patient ones. As standard, the solution to (1), denoted by $(\hat{x}_1, \hat{x}_2)$ satisfies the first-order condition $u'\left(x_1\right) = \underline{R}u'\left(x_2\right)$ and (2). At $(\hat{x}_1, \hat{x}_2)$, (3) does not bind.

Unlike much of the literature following Diamond-Dybvig (1983), we do not impose the assumption that the coefficient of relative risk aversion (CRRA), defined as $-xu''/u'$, is greater than 1. The inverse of CRRA, $-u'/xu''$, is the elasticity of intertemporal substitution (EIS). A higher CRRA means that depositors are more risk averse. It also means the EIS is low. The assumption that CRRA $> 1$ results in $1 < \hat{x}_1 < \hat{x}_2 < \underline{R}$, meaning the depositors prefer a smoother consumption profile than what they can have in autarky. However, if $0 <$ CRRA $< 1$, the contract specifies $\hat{x}_1 < 1 < \underline{R} < \hat{x}_2$, indicating that depositors prefer a more volatile consumption profile than under autarky. This does not mean depositors are not risk averse. They are. However, since the technology yields a higher return, the depositors prefer to substitute away from early consumption toward higher late consumption.

Without the loan market, borrowers solve:

$$\hat{W}_B = \max_k \left[-c(k) + v\left(\bar{R}k\right)\right]$$

The first-order condition is $c'(k) = \bar{R}v'\left(\bar{R}k\right)$. Let the solution for capital be denoted by $\hat{k}$. Borrowers consume $\hat{x}_B = \bar{R}\hat{k}$.

## 2.3 Competitive Loan Market

Next, we set up the problem in a competitive loan market. Both banks and borrowers take the market loan rate as given and choose the size of the loan to maximize their expected utility. The bank's problem is

$$\begin{aligned} &\max_{x_1, x_2, a} \left[\lambda u(x_1) + (1-\lambda) u(x_2)\right] \\ \text{s.t.}\ \ &(1-\lambda) x_2 = (1 - \lambda x_1 - a) r + a\underline{R} \end{aligned} \tag{4}$$

and (3), where $r$ is the market loan rate. The bank keeps $\lambda x_1$ in storage and invests $a$ in its own long-term technology.[9] Thus, $1 - \lambda x_1 - a$ is the amount of the loan extended to the borrowers. Equation (4) is the resource constraint for the bank. The bank expects the loan to be paid at $t = 2$ with the interest rate $r$. From its own production, the bank pays impatient depositors $\lambda x_1$ at $t = 1$ and the safe-haven investment $a$, matures with return $\underline{R}$. With these resources, the bank pays the patient depositors at $t = 2$.

The first-order condition with respect to $x_1$ is

$$u'(x_1) - ru'(x_2) = 0 \tag{5}$$

By (4), $a = 0$ if $r > \underline{R}$, $0 < a < 1 - \lambda x_1$ if $r = \underline{R}$, and $a = 1 - \lambda x_1$ if $r < \underline{R}$. Next, take the

[9]It does not matter whether the bank keeps $\lambda x_1$ in storage or in its own long-term technology because these two options yield the same return at $t = 1$.

derivative of (5) with respect to $r$ for $r > \underline{R}$, we get the slope of the loan supply curve as

$$\frac{d\ell^s}{dr} = \frac{d\left(1 - \lambda x_1\right)}{dr} = \frac{-\lambda\left[u'(x_2) + x_2 u''(x_2)\right]}{u''(x_1) + \frac{\lambda r^2 u''(x_2)}{1-\lambda}} \tag{6}$$

where $\ell^s$ denotes the supply of loans. The denominator is negative and the numerator depends on the intertemporal marginal rate of substitution at $x_2$. With the assumption that EIS $< 1$ in the literature following Diamond and Dybvig (1983), the numerator is positive. Hence, the supply of loans, $1 - \lambda x_1$, decreases in $r$. If EIS $> 1$, the numerator of (6) is negative, which means the supply of loans is increasing in $r$. A more general case is that EIS moves above and below 1 as $x_2$ changes, so the supply curve is non-monotone. Figure 1 shows two monotonic cases of the loan supply curve for $\underline{R} < r \leq \bar{R}$. The vertical segment of the supply curve represents the quantities at which depositors are indifferent, as the return on the loan is the same as that of their own safe investment.

The borrower's problem is

$$\max_{k,\ell,x_B}\left[-c\left(k\right) + v\left(x_B\right)\right]$$

$$\text{st } x_B = \left(\bar{R} - r\right)\ell + \bar{R}k \tag{7}$$

$$x_B \geq \left(1 - \chi\right)\left(\ell + k\right) \tag{8}$$

where $\ell$ is the loan amount. Equation (7) is the borrower's resource constraint. His consumption is financed by two sources: he borrows $\ell$ from the bank, invests it, gets a return of $\bar{R}$ and pays $r$; in addition, his own capital yields $\bar{R}$. The borrower's repayment constraint is described by (8). The borrower can abscond with $1 - \chi$ fraction of the investment, and the bank recovers $\chi$ fraction of it. The contract must ensure that the borrower's equilibrium payoff cannot be less than the deviation payoff.

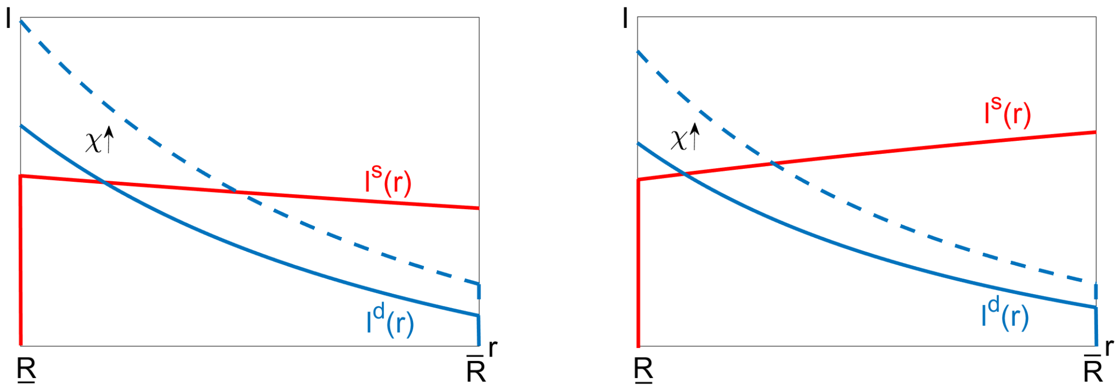
The first-order conditions are

$$-\left(r - \bar{R} + 1 - \chi\right)c'\left(k\right) + r\left(1 - \chi\right)v'\left(x_B\right) = 0 \tag{9}$$

$$\eta - \frac{\left(\bar{R} - r\right)v'\left(x_B\right)}{r - \bar{R} + 1 - \chi} = 0 \tag{10}$$

Figure 1: Competitive Loan Market Equilibrium

(a) Downward-sloping $\ell^s$, $-u'/xu'' < 1$ (b) Upward-sloping $\ell^s$, $-u'/xu'' > 1$




*Note:* The figure illustrates equilibrium in the competitive loan market. The red curve represents the supply of loans from banks, $\ell^s(r)$. The left panel shows that when the elasticity of intertemporal substitution (EIS) $< 1$, the supply of loans decreases in the loan rate $r$. The right panel shows that when EIS $> 1$, loan supply increases in $r$. The vertical segment of the supply curve is the range of loan quantities for which banks are indifferent between lending and investing in their own safe technology yielding return $\underline{R}$. The blue curve represents the demand for loans by borrowers, $\ell^d(r)$, which is downward sloping in both panels. The blue dashed curve is the upward shift in the original loan demand curve when pledgeability $\chi$ increases.

where $\eta$ is the Lagrangian multiplier associated with (8); in other words, the shadow value of the borrower's repayment constraint. Note that for $r < \bar{R} - 1 + \chi$, there will be infinite demand for $\ell$. So $r$ cannot be lower than $\bar{R} - 1 + \chi$. There are two cases, depending on whether (8) binds. First, with $r = \bar{R}$, then $k = \hat{k}$ and $\ell < \left(\bar{R} - 1 + \chi\right)\hat{k}/(1-\chi)$. This is represented by the vertical part of the blue curve in Figure 1. Second, with $\bar{R} - 1 + \chi \leq r < \bar{R}$, the demand for loans is solved from (9) with binding (8). That is,

$$\left(r - \bar{R} + 1 - \chi\right) c'\left(\frac{r - \bar{R} + 1 - \chi}{\bar{R} - 1 + \chi}\ell\right) = r\left(1-\chi\right) v'\left(\frac{r\left(1-\chi\right)}{\bar{R} - 1 + \chi}\ell\right) \tag{11}$$

Taking the derivative of (11) with respect to $r$, we get the slope of the loan demand curve:

$$\frac{d\ell^d}{dr} = \frac{-\frac{\left(\bar{R}-1+\chi\right)^2}{r}c' - \left(r - \bar{R} + 1 - \chi\right)\ell c'' + (1-\chi) r \ell v''}{\left(r - \bar{R} + 1 - \chi\right)^2 c'' - r^2 (1-\chi)^2 v''} \tag{12}$$

As $c', c'' > 0 > v''$, (12) is negative. Hence, the demand for loans decreases in $r$, as shown in Figure 1.

Taking the derivative of (11) with respect to $\chi$, we get

$$\frac{d\ell^d}{d\chi} = \frac{-\frac{(\bar{R}-r)(\bar{R}-1+\chi)}{1-\chi}c' - rkc'' + x_B r\bar{R}v''}{r^2(1-\chi)^2 v'' - \left(r - \bar{R} + 1 - \chi\right)^2 c''}$$

which is positive. This implies that when $\chi$ increases, the demand curve shifts up. See the blue dashed lines in Figure 1.

Finally, the loan-market clearing condition is

$$1 - \lambda x_1 - a = n\ell \tag{13}$$

which solves for equilibrium $r$.

# 3 Results

## 3.1 Theoretical Results

There are three cases for equilibrium contracts, depending on whether eq. (8) binds and whether $a > 0$ or $a = 0$. We refer to the High-pledgeability Region as the range of $\chi$ values where eq. (8) does not bind, the Medium-pledgeability Region where eq. (8) binds and $a = 0$, and the Low-credibility Region where eq. (8) binds and $a > 0$.

If the loan supply curve is upward sloping, it is obvious that the intersection of $\ell^s$ and $\ell^d$ is unique (see Figure 1b). An increase in $\chi$ shifts up $\ell^d$ but does not affect $\ell^s$. For a high value $\chi$, $\ell^d$ and $\ell^s$ intersect at $r = \bar{R}$, where (8) does not bind. For low value $\chi$, they intersect at $r = \underline{R}$, where (8) binds and $a > 0$. For intermediate value, they intersect at $r$ between $\underline{R}$ and $\bar{R}$, where (8) binds and $a = 0$. Hence, there are two cutoffs of $\chi$, labeled $\bar{\chi}$ and $\underline{\chi}$, $\bar{\chi} > \underline{\chi}$.

However, when $\ell^s$ is downward sloping or nonmonotone, it is less clear if the equilibrium is unique and if there are only two cutoffs. In Appendix A, we prove that

$\ell^s$ is flatter than $\ell^d$ in the Medium Region whenever the two curves intersect. Thus, $\ell^s$ and $\ell^d$ cross at most once in the Medium Region, and the equilibrium is indeed unique. The upshot is that there are two cutoffs for three distinct regions. In any case, we have the following proposition.

**Proposition 1** *Consider a perfectly competitive loan market. There exist $\underline{\chi}$ and $\bar{\chi}$, with $\underline{\chi} < \bar{\chi}$ such that (1) if $\chi \geq \bar{\chi}$, the equilibrium is in the High-pledgeability Region; (2) if $\underline{\chi} \leq \chi < \bar{\chi}$, the equilibrium is in the Medium-pledgeability Region; (3) if $\chi < \underline{\chi}$, the equilibrium is in the Low-pledgeability Region.*

The proof is provided in Appendix A. We discuss each case in detail below.

**High-Pledgeability Region:** With $\eta = 0$, we solve (4)-(13) to get $a = 0$, $r = \bar{R}$, $k = \hat{k}$, $x_1 = x_1^*$ and $x_2 = x_2^*$, where $\left(x_1^*, x_2^*\right)$satisfies $u'(x_1) = \bar{R}u'(x_2)$ and $(1 - \lambda x_1)\bar{R} = (1 - \lambda)x_2$. The aggregate loan size is $1 - \lambda x_1^*$. Each borrower gets $\ell^* = \left(1 - \lambda x_1^*\right)/n$. The economy is in this region if and only if $\chi \geq \bar{\chi} \equiv 1 - \hat{k}\bar{R}/\left(\ell^* + \hat{k}\right)$.

In the High-pledgeability Region, the bank takes full advantage of the borrower's technology. The marginal rate of substitution between $x_1$ and $x_2$ equals the marginal rate of transformation of the borrower's technology. The borrower's capital production and consumption are the same as in autarky.

**Medium-Pledgeability Region:** With $\eta > 0$ and $a = 0$, the Medium Region is associated with values of $\underline{\chi} \leq \chi < \bar{\chi}$, where $\underline{\chi}$ is defined in the proof of Proposition 1 in the Appendix. In this region, $\underline{R} \leq r < \bar{R}$.

In the Medium Region, $r$ and $\chi$ are positively related. When $\chi$ increases, borrowers are able to borrow more, which intensifies competition for loans. This competition drives up the interest rate. Whether the loan size increases depends on the slope of $\ell^s$. If $\ell^s$ is downward sloping, the equilibrium loan size decreases with $\chi$, and vice versa.

It is ambiguous whether borrowers produce more capital compared to the High-pledgeability Region. Two countervailing forces are at work. On the one hand, as assets become less pledgeable, the loan contract requires more collateral to induce the

Table 1: Competitive Market: Comparative Statics

| | $dx_1/d\chi$ | $dx_2/d\chi$ | $dx_B/d\chi$ | $dk/d\chi$ | $dr/d\chi$ | $d\ell/d\chi$ | $da/d\chi$ |
|---|---|---|---|---|---|---|---|
| High | 0 | 0 | 0 | 0 | 0 | 0 | N.A. |
| Medium ($-u'/xu'' < 1$) | + | + | ± | ± | + | − | N.A. |
| Medium ($-u'/xu'' > 1$) | − | + | ± | ± | + | + | N.A. |
| Low | 0 | 0 | + | ± | 0 | + | − |

*Note:* The table reports the comparative statics of the equilibrium with respect to pledgeability $\chi$ in competitive loan and deposit markets. The three rows correspond to the High-, Medium-, and Low-pledgeability Regions characterized in Proposition 1. In the Medium Region, the sign of several comparative statics depends on the depositor's elasticity of intertemporal substitution, $-u'/xu''$. The signs +, −, and 0 indicate whether the variable increases, decreases, or remains unchanged as $\chi$ increases. The symbol ± indicates that the sign is ambiguous and depends on parameter values, while N.A. denotes that the variable is not defined in that region.

borrower to repay the loan. On the other hand, since $r$ is lower in the Medium Region, the unit repayment cost is smaller, which helps relax the repayment constraint.

We find that, when the depositor's EIS is low, the borrower's EIS, $-v'/xv''$, determines the sign of the overall effect. With a high elasticity, borrowers reduce the production of collateral as the opportunity cost becomes higher when $r$ increases; the opposite occurs when elasticity is low. However, when depositor's elasticity is high, the overall effect is indeterminate. For the same reason, $x_B$ is not necessarily monotone in $\chi$.

**Low-Pledgeability Region:** With $\eta > 0$ and $a > 0$, we have that $r = \underline{R}$. It follows immediately that $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$, as in autarky. Borrowers take full advantage of the loan market as they pay the bank's reservation rate. This region arises when $\chi < \underline{\chi}$. In the Low-pledgeability Region, $\ell$ and $x_B$ are both strictly increasing in $\chi$, while $a$ is strictly decreasing. Again, we find that whether $k$ increases or decreases depends on $-v'/xv''$.

The comparative statics for each of the three regions are summarized in Table 1. In the Low Region, the sign of $dk/d\chi$ follows $dk/d\chi \doteq -(v'/v''x + 1)$, where $\doteq$ indicates that the derivative has the same sign as the expression on the right-hand side. In the Medium Region, if $u'/u''x + 1 > 0$, then $dk/d\chi \doteq v'/v''x + 1$.

Consider a case in which $\chi$ increases from 0. The borrowing conditions change in several dimensions: collateral, loan rate, and loan size. Note that with $\chi = 0$, banks can still lend some positive amount to borrowers because the borrowers would have to give up the return on their own capital if they abscond. As $\chi$ increases, the borrowers become less constrained and borrow more. Banks engage less in direct investment, making more loans. However, the loan rate stays at $\underline{R}$ because the demand for loans is still too low to move the rate. As $\chi$ crosses $\underline{\chi}$, banks no longer invest in the low-return technology. The competition for loans eventually raises the loan rate. Whether loan size rises or falls with $\chi$ then depends on the elasticity of loan supply. As $\chi$ increases further and crosses $\bar{\chi}$, the repayment constraint no longer binds. The loan rate stays at $\bar{R}$, and the loan size stays constant. Throughout these adjustments, the loan rate also affects capital production and deposit rates. Borrowers' and depositors' EIS determine how they respond to the higher loan return. Specifically, whether they prefer to smooth consumption over time or accept a more uneven consumption profile to take greater advantage of that return.

### 3.2 Numerical Results

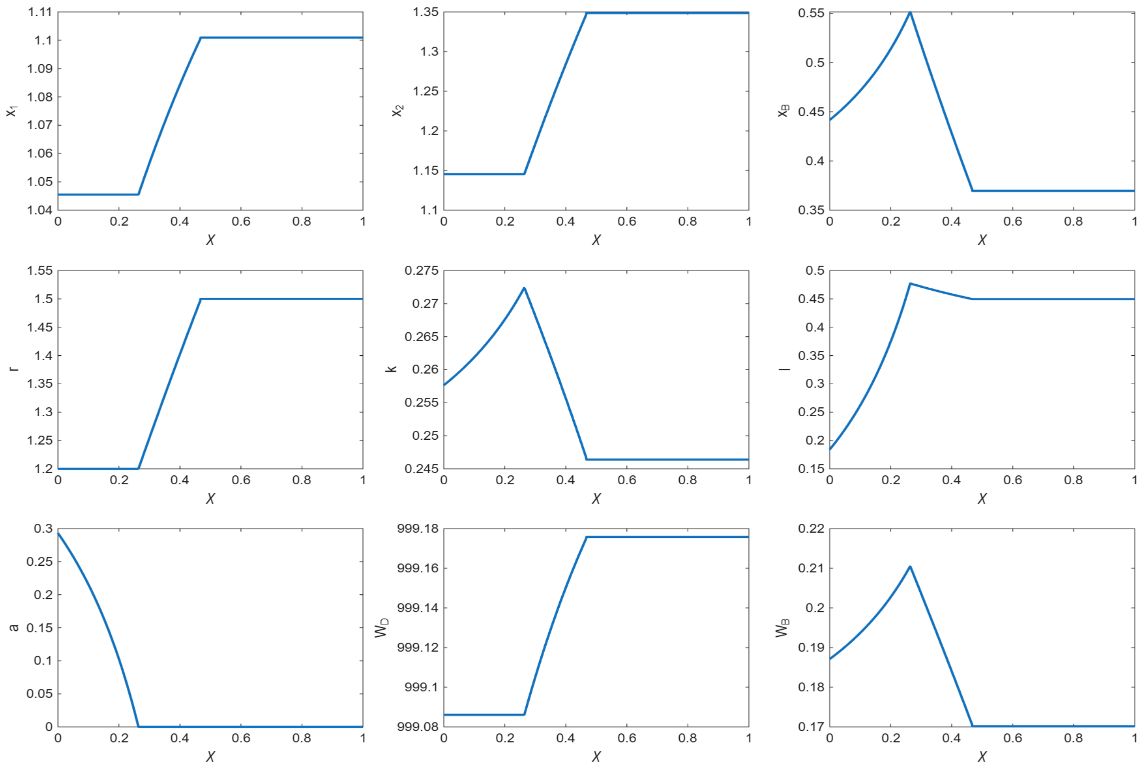
To illustrate the equilibrium outcomes for different values of collateral pledgeability, we turn to numerical analysis. The following functional forms and parameter values are used in the numerical experiments. Let

$$u(x) = \frac{(x+b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma}, \quad c(k) = Bk^{\alpha}, \quad v(x) = A\frac{(x+b)^{1-\delta} - b^{1-\delta}}{1-\delta},$$

where $b = 0.001$, $\gamma = 2$, $B = 1$, $\alpha = 2$ and $A = 0.2$. Other parameters are $\bar{R} = 1.5$, $\underline{R} = 1.2$, $\lambda = 0.5$, and $n = 1$. Note that Figure 2 plots the combinations of variables and $\chi$ with $\delta = 0.5$ while Figure 3 plots the combinations with $\delta = 2$.

The parameters $\gamma$ and $\delta$ determine the EIS of depositors and borrowers, respectively. If $\gamma > 1$, depositor's EIS $< 1$. If $\gamma < 1$, their EIS $> 1$. The same relationship applies to borrowers through $\delta$. Given $\gamma > 1$, the adjustment in $k$ depends on the borrower's

Figure 2: Competitive Equilibrium ($\gamma = 2, \delta = 0.5$)

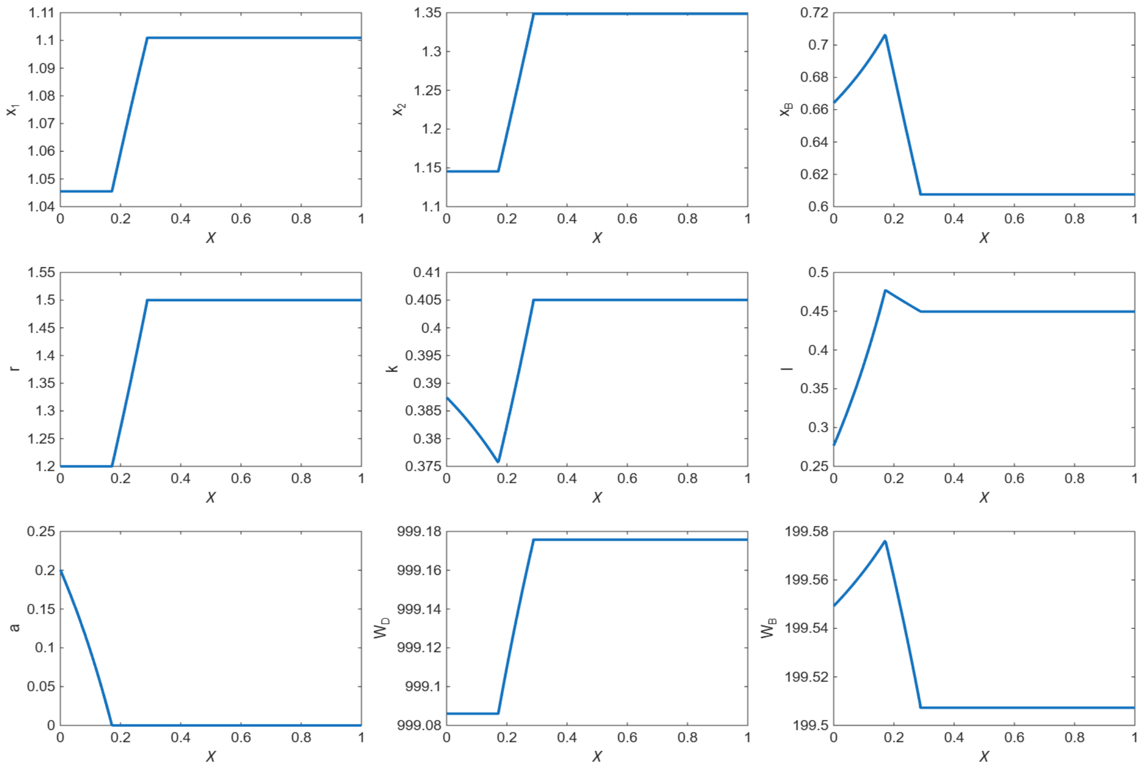


*Note:* The figure shows equilibrium outcomes as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 2$. In this case, the borrower's EIS $\delta = 0.5$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

EIS. With $\delta = 0.5$, capital initially increases in the Low Region and then decreases in the Medium Region (Figure 2). Conversely, when $\delta = 2$, the adjustment follows the opposite pattern (Figure 3).

In both examples, an increase in $\chi$ raises loan size, borrowers' consumption, and borrowers' welfare in the Low-pledgeability Region. These relationships reverse in the Medium-pledgeability region and become invariant to increases in $\chi$ in the High-pledgeability region. In the example of Figure 3, collateral intensity (i.e., $k/\ell$ ratio) rises as assets become more pledgeable in the Medium Region. The loan rate is constant in the Low and High Regions and strictly increasing in the Medium Region.

Figure 3: Competitive Equilibrium ($\gamma = 2, \delta = 2$)

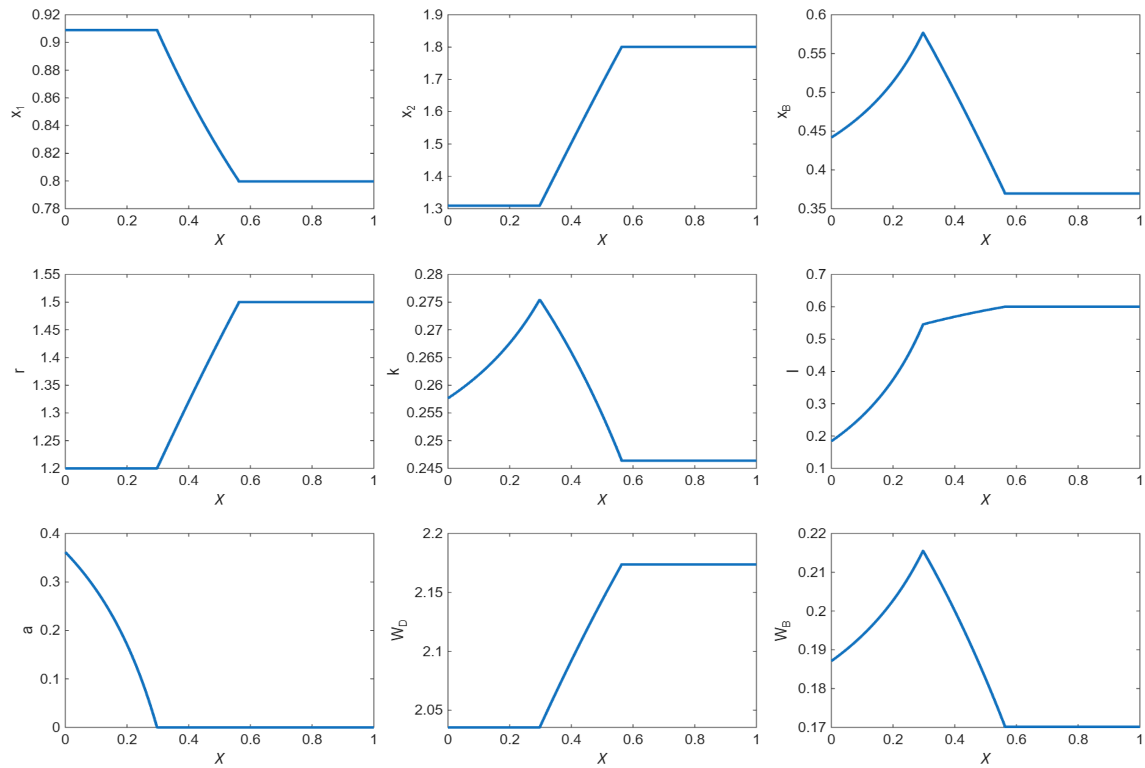



*Note:* The figure shows equilibrium outcomes as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 2$. Here, borrower's EIS $\delta = 2$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

The last row of Figure 2 shows that borrowers can in fact be worse off as pledgeability increases. In the Medium Region, borrowers are worse off due to smaller loans. In contrast, lenders (depositors) are better off as loan rates increase with pledgeability. Indeed, the top row of Figure 2 shows that the consumption of both impatient and patient depositors increases with pledgeability.

On the deposit side, both types of depositors consume more as $\chi$ increases, but patient depositors' consumption $x_2$ increases faster than impatient depositors' consumption $x_1$. As a result, risk sharing is weaker, but the overall welfare of depositors $W_D$ improves.

Figure 4: Competitive Equilibrium ($\gamma = 0.5, \delta = 0.5$)

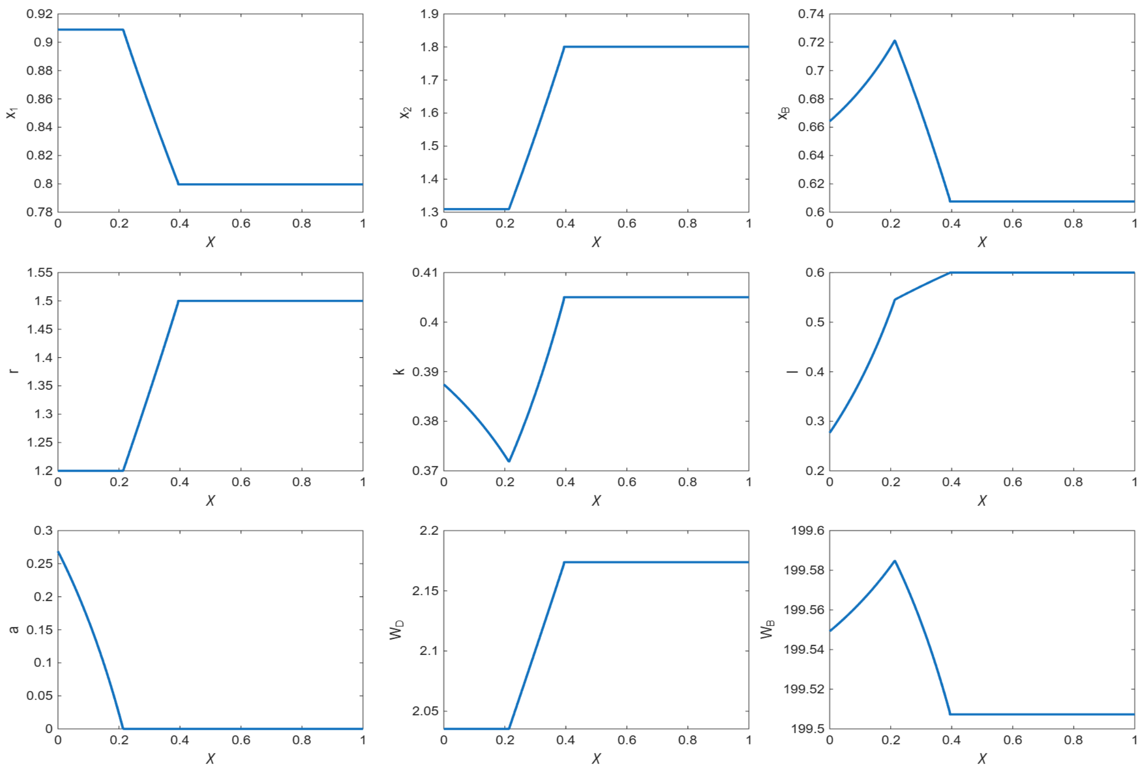


*Note:* The figure shows equilibrium outcomes as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 0.5$, and the borrower's EIS is $\delta = 0.5$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

Figures 4 and 5 plot examples using the same parameter values as in Figures 2 and 3, except that $\gamma = 0.5$. As $\chi$ increases, depositors consume less $x_1$ and more $x_2$ in the Medium Region, and the loan size increases monotonically across Regions.[10] So, risk sharing decreases in pledgeability. The Medium-pledgeability region shows how the

[10]See Drechsler, Savov and Schnabl (2021). They consider a model in which banks have market power in the deposit market, whereas we focus on limited commitment in the loan market. Drechsler, et al. use the federal funds rate as a measure of the risk-free rate. Banks face two separate costs; one is interest-insensitive franchise costs and the other is interest-sensitive deposit expenses. The bank's asset allocation minimizes costs subject to a solvency condition. In their model, market power is sufficient to account for their empirical finding: namely, there is sufficient deposit supply such that the bank's deposit rates increase about one-third the size of the increase in short-term market rates (see Table 1). Dreschler, et al.'s results raise the question about the empirical validity of EIS less one. The risk-free-to-deposit rate spread is not monotonically related to changes in pledgeability in our framework, suggesting that future work could explore how this limited commitment affects the deposit channel.

Figure 5: Competitive Equilibrium ($\gamma = 0.5, \delta = 2$)




*Note:* The figure shows equilibrium outcomes as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 0.5$ and the borrower's EIS to $\delta = 2$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

positive relationship between loan rate and pledgeability adversely affects borrowers. Capital gains from borrowing are diminishing as pledgeability improves, thus reducing borrower's consumption, regardelss of the borrower's EIS.

Our numerical experiments are not intended as a kind of quantitative identification. In particular, the cut-off points for the different regions should not be interpreted literally. Nevertheless, even this simple numerical exercise provides useful qualitative insights.

To compare our results with Kaplan and Zingales (1997), we use the marginal cost of producing $k$ as the rate of internal financing and the loan rate $r$ as the external financing rate. The difference, $c' - r$, increases in the Low Region and decreases in the Medium Region in the numerical examples above with $\delta = 0.5$, whereas it decreases in both regions when $\delta = 2$. This shows that interest rate differentials are not reliable indicators of underlying frictions. Even if other contract terms are nonmonotonic in interest rate differentials, this does not imply that they are nonmonotonic in the underlying frictions.

The non-monotonicity results are robust to alternative market structures where banks have market power, modeled through bilateral trading with Nash bargaining or directed search (see Appendix B). The underlying mechanism differs from that in the competitive equilibrium: agents can adjust multiple elements of the contracts, each operating on different margins when the repayment constraint binds. Thus, changes in pledgeability do not necessarily cause all terms to move uniformly in magnitude or even direction. Despite these complexities, a common feature emerges across market structures: investment in the low-return, safe asset falls with pledgeability in the Low-pledgeability Region and becomes zero once the economy enters the Medium Region. In other words, using the low-return technology is the last resort under the worst credit conditions. This is because the production frontier is invariant to small changes in contract terms, whereas investing in the low-return asset shrinks the production set.

# 4 Financial Stability

We extend the model in Section 2.3 to include stochastic returns and defaults. Suppose there are two realizations of the aggregate production state. With probability $p_H$, the economy is in State $H$, and the borrowers' production yields $\bar{R} > \underline{R}$. With probability $p_L = 1 - p_H$, the economy is in State $L$ and the return is 0. In State $L$, borrowers cannot repay the loan and thus default happens. Assume $p_H\bar{R} > \underline{R}$, so making some investment in the risky technology is potentially optimal. If a borrower reneges, the return in State $H$ is 1, but in State $L$, it is 0. This is to ensure that reneging is not socially desirable in either state.

As a first step, Section 4.1 characterizes the optimal contracts given that the aggregate state is not revealed until $t = 2$. The bank provides a demand deposit contract and allocates investment, taking into account that default happens with positive probability. In Section 4.2, we consider the situation in which depositors have private information about the production state in $t = 1$ and can make early withdrawals if default is expected. Anticipating depositors' choice, banks may offer a run-proof contract that guarantees late payments are bigger than early payments (Section 4.2.1), or a run-admitting contract that fails to guarantee such a promise in state $L$ (Section 4.2.2). Our goal is to show how pledgeability in the loan market can affect the bank's choice of contract and financial stability. Because our focus is on how pledgeability affects banking crises, we do not consider panic-driven bank runs.

## 4.1 No Private Information

Suppose the aggregate state is revealed to all agents only in $t = 2$. Banks maximize depositors' ex ante expected utility, and depositors withdraw according to their consumption timing. Because borrowers can repay only in State $H$, their problem becomes

$$\max_{k,\ell,x_B} \left[-c\left(k\right) + p_H v\left(x_B\right)\right]$$
$$\text{st (7) and (8)}$$

which yields two FOCs

$$\begin{aligned} -\left(r - \bar{R} + 1 - \chi\right) c'\left(k\right) + p_H r \left(1 - \chi\right) v'\left(x_B\right) &= 0 \qquad (14) \\ \eta - \frac{p_H v'\left(x_B\right)\left(\bar{R} - r\right)}{r - \bar{R} + 1 - \chi} &= 0 \end{aligned}$$

similar to (9) and (10).

The bank provides a state-contingent contract and allocates deposits between the safe technology and the loan market. Under the modified setup, the deposit contract solves

$$\max_{x_1, x_{2,L}, x_{2,H}, a} \{\lambda u(x_1) + (1-\lambda)[p_L u(x_{2,L}) + p_H u(x_{2,H})]\} \tag{15}$$

$$\text{st } x_{2,L} = \frac{a\underline{R}}{1-\lambda} \text{ and } x_{2,H} = \frac{(1-\lambda x_1 - a) r + a\underline{R}}{1-\lambda}$$

$$p_L u(x_{2,L}) + p_H u(x_{2,H}) \geq u(x_1) \tag{16}$$

where $x_{2,L}$ and $x_{2,H}$ denote the payment in State $L$ and $H$ in period 2, respectively. The first two constraints are the bank's state-contingent resource constraints. After paying $\lambda x_1$ to early depositors, the bank invests $a$ in safe technology and $1 - \lambda x_1 - a$ in the loan market. In state $L$, borrowers default, so the period-2 resources consist only of the return $a\underline{R}$ from safe technology. In state $H$, the bank also receives the loan repayment $(1 - \lambda x_1 - a)r$. Because the mass of late depositors is $1 - \lambda$, dividing the available resources by $1 - \lambda$ gives $x_{2,L}$ and $x_{2,H}$. The final constraint ensures that depositors prefer waiting until period 2 to withdrawing early. We assume $p_L$ is small, so in equilibrium (16) it does not bind.

We first show that, under the assumption that $p_H \bar{R} > \underline{R}$, investing all remaining funds in the safe technology, i.e., setting $a = 1 - \lambda x_1$, is not optimal.

**Lemma 1** *Given $p_H \bar{R} > \underline{R}$, the solution to (15) must have $a < 1 - \lambda x_1$.*

**Proof.** See Appendix A. ■

The bank's first-order condition with respect to $x_1$ is

$$u'(x_1) - p_H u'\left(\frac{(1-\lambda x_1 - a) r + a\underline{R}}{1-\lambda}\right) r = 0 \tag{17}$$

Since Lemma 1 rules out $a = 1 - \lambda x_1$, the FOC with respect to $a$ is

$$p_L u'\left(\frac{a\underline{R}}{1-\lambda}\right) \underline{R} - p_H u'\left(\frac{(1-\lambda x_1 - a) r + a\underline{R}}{1-\lambda}\right) \left(r - \underline{R}\right) \leq 0 \tag{18}$$

which holds with equality if $a > 0$. Note that $r = \underline{R}$ cannot satisfy (18). The loan rate must be higher than $\underline{R}$ to compensate for loan risk. We have the following lemma to show the relationship between $x_1$ and $x_{2,L}$.

**Lemma 2** *The solution to (15) has $x_{2,L} > x_1$ if and only if $r < \underline{R}/\left(1 - p_L \underline{R}\right)$.*

**Proof.** See Appendix A. ■

With the market clearing condition (13), we can solve for the equilibrium and categorize it as follows.

(1) Non-Binding Repayment Constraint Region: In this region, $\eta = 0$, implying that $\bar{R} = r$ and $c'(\hat{k}) = p_H v'(\bar{R}\hat{k})\bar{R}$. There are two cases, depending on whether $a > 0$.

(Condition 1a) If

$$p_L u'(0)\,\underline{R} - p_H u'\left(\frac{1 - \lambda \hat{x}_1}{1 - \lambda}\bar{R}\right)\left(\bar{R} - \underline{R}\right) \leq 0$$

where

$$u'(\hat{x}_1) - p_H u'\left(\frac{1 - \lambda \hat{x}_1}{1 - \lambda}\bar{R}\right)\bar{R} = 0$$

then $a = 0$, $x_1 = \hat{x}_1$.

(Condition 1b) If

$$p_L u'(0)\,\underline{R} - p_H u'\left(\frac{1 - \lambda \hat{x}_1}{1 - \lambda}\bar{R}\right)\left(\bar{R} - \underline{R}\right) > 0 \tag{19}$$

then $a > 0$ and $(a, x_1)$ solves

$$\begin{aligned} u'(x_1) - p_H u'\left(\frac{(1 - \lambda x_1 - a)\,\bar{R} + a\underline{R}}{1 - \lambda}\right)\bar{R} &= 0 \\ p_L u'\left(\frac{a\underline{R}}{1 - \lambda}\right)\underline{R} - p_H u'\left(\frac{(1 - \lambda x_1 - a)\,\bar{R} + a\underline{R}}{1 - \lambda}\right)\left(\bar{R} - \underline{R}\right) &= 0 \end{aligned}$$

which are independent of $\chi$.

Under Condition 1b, the borrower's repayment constraint does not bind when $\chi \geq \bar{\chi} \equiv 1 - \hat{k}\bar{R}/[(1 - \lambda x_1 - a)/n + \hat{k}]$. Although the loan rate is at its highest, depositors do not necessarily make all long-term investment in the borrower's technology. Precaution can account for the bank investing some deposits in the safe asset because of the default risk.

(2) Binding Repayment Constraint Region: With $\eta > 0$ and (18), the loan rate is $r \in (\underline{R}, \bar{R})$. The contract terms are jointly determined by (17)-(18) and the market-clearing condition (13).

Default risk complicates the bank's portfolio choice because the bank needs to balance the intertemporal and risk-related tradeoffs. Compared to the contracts in the nonstochastic setting in Section 2.3, the Binding Repayment Constraint Region does not offer clear results on when the bank will invest in safe technology. In other words, we do not have a definitive answer regarding whether there exists a unique cutoff $\underline{\chi}$ such that $a = 0$ above the cutoff and $a > 0$ below it. Nor can we guarantee that the equilibrium is unique in this Region.[11]

We therefore resort to numerical examples to illustrate the patterns. In the examples, the equilibrium is unique across the entire range of $\chi$. Figures 6 and 7 plot contracts using the parameters from Figures 2 and 3, respectively. The probability of State H is $p_H = 0.999$. In both examples, $a > 0$ on the entire range. As $\chi$ increases, the economy switches from the Binding Region to the Nonbinding Region. The contract terms exhibit patterns similar to the benchmark model but are smoother. In the Binding Region, $k$, $\ell$, $x_B$, and $W_B$ exhibit nonmonotonicity. This is not caused by a switch between regions; rather, it arises because $\chi$ affects $k$ through two opposing channels, as discussed in Section 3. On the one hand, an increase in $\chi$ relaxes the borrowing constraint. On the other hand, increased competition for loans raises $r$, thereby reducing loan demand.

[11] Using the first-order conditions from the borrower's side, the demand for loan can be derived as

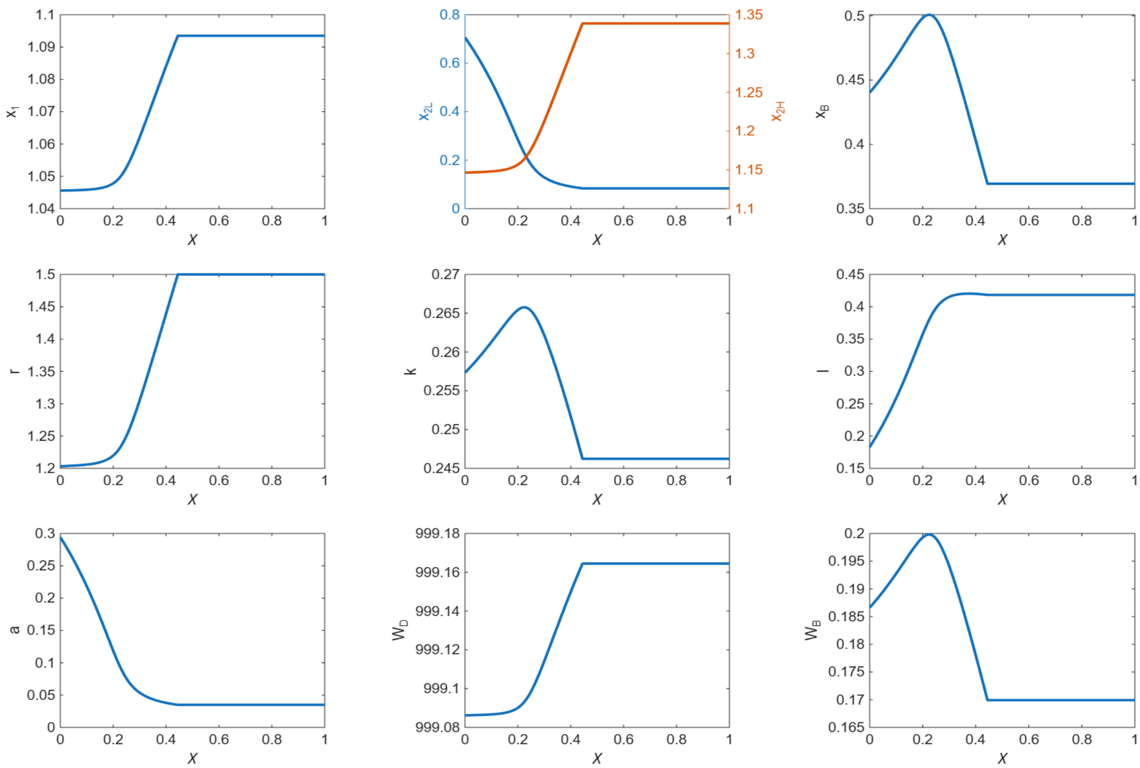
$$\frac{d\ell^d}{dr} = \frac{-\frac{\bar{R}-1+\chi}{r}c' - c''k + p_H(1-\chi)v''x_B}{\frac{\left(r-\bar{R}+1-\chi\right)^2}{\bar{R}-1+\chi}c'' - p_H\frac{r^2(1-\chi)^2}{\bar{R}-1+\chi}v''}$$

which is negative. Assume $a = 0$. The slope of loan supply is

$$\frac{d\ell^s}{dr} = p_H\frac{\left(p_L u_L''\frac{\lambda\underline{R}^2}{1-\lambda} + u_1''\right)\left[u_H' + u_H''\frac{1-\lambda x_1-a}{1-\lambda}\left(r-\underline{R}\right)\right] + u_1'u_H''\frac{\lambda}{1-\lambda}\underline{R}\left[1-p_H\left(r-\underline{R}\right)\right] + p_L u_L''\frac{\lambda\underline{R}^2}{1-\lambda}u_H''\frac{1-\lambda x_1-a}{1-\lambda}\underline{R}}{u_1''\left[p_L u_L''\frac{\underline{R}^2}{1-\lambda} + p_H u_H''\frac{\left(r-\underline{R}\right)^2}{1-\lambda}\right] + p_H u_H''\frac{\lambda r^2}{1-\lambda}p_L u_L''\frac{\underline{R}^2}{1-\lambda}},$$

which has an ambiguous sign. If $p_L \to 0$, by the argument in the proof of Proposition 1 and by continuity, $d\ell^s/dr > d\ell^d/dr$, and the equilibrium is unique. However, we cannot determine the sign in general.

Figure 6: Competitive Equilibrium, Stochastic Return ($\gamma = 2, \delta = 0.5$)




*Note:* The figure shows equilibrium outcomes as collateral pledgeability, $\chi$, varies in the model with stochastic returns but no private information. The depositors' EIS parameter is set to $\gamma = 2$, and the borrower's to $\delta = 0.5$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption in state L $x_{2,L}$, patient depositors' consumption in state H $x_{2,H}$ (orange line), borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

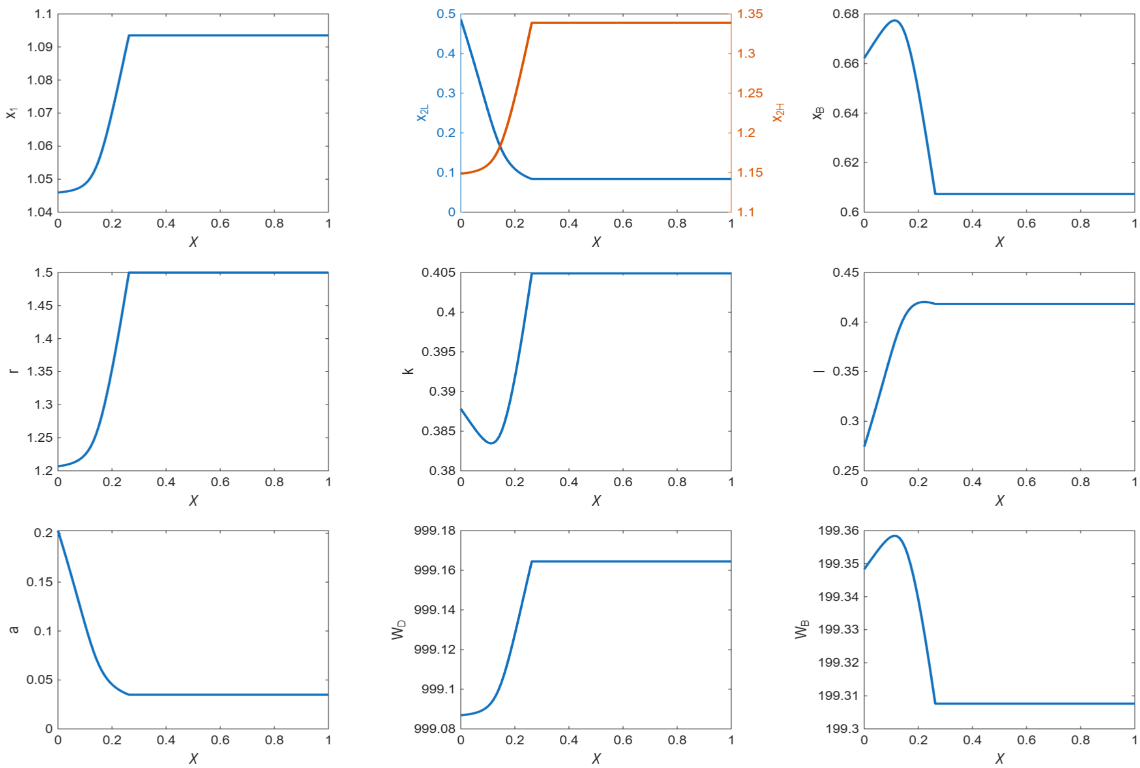
Consequently, the overall effect on $k$ is ambiguous.

Also note that in these examples $x_{2,L} < x_1$. If depositors received private information at $t = 1$ about the loan return, there would be a bank run in State $L$. Our next task is to find the optimal demand deposit contract in this situation.

## 4.2 Private Information and Bank Runs

We modify the model setup as follows. Depositors are informed of the state at $t = 1$. If the state is $L$, they know that borrowers will default and that period-2 payments can be financed only by the returns on the bank's safe investment. We follow Cooper and

Figure 7: Competitive Equilibrium, Stochastic Return ($\gamma = 2, \delta = 2$)




*Note:* The figure shows equilibrium outcomes as collateral pledgeability, $\chi$, varies in the model with stochastic returns but no private information. The depositors' EIS parameter is set to $\gamma = 2$, and the borrower's to $\delta = 2$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption in state L $x_{2,L}$, patient depositors' consumption in state H $x_{2,H}$ (orange line), borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

Ross (1998) to formulate the optimal demand deposit contract without suspension of convertibility.

In State $L$, depositors compare $x_1$ and $x_{2,L}$ to decide when to withdraw. If $x_1 \leq x_{2,L}$, patient depositors prefer to wait until the last period, conditional on the belief that all other patient depositors will also wait. Since we rule out panic runs, no bank run occurs in this scenario. Also note that if $x_1 = x_{2,L}$, patient depositors are indifferent between withdrawing at $t = 1$ and $t = 2$, and we assume that they withdraw at $t = 2$. If $x_1 > x_{2,L}$, however, all depositors will withdraw in $t = 1$ regardless of others' strategies, and the bank will be short of liquidity and fail.

Anticipating depositors' withdrawal decisions, the bank can choose from two classes of contracts. The first class, which we refer to as *run-proof contracts*, provides $x_1 \leq x_{2,L}$. This requires the bank to make sufficient investments in safe technology so that, even in the state where borrowers default, the bank still has enough resources to repay depositors. The second class, which we refer to as *run-admitting contracts*, satisfies $x_1 > x_{2,L}$. Under such a contract, the bank allocates more funds to the loan market, creating a more volatile consumption profile in $t = 2$. In what follows, we formulate the contract in each category. The optimal contract is the better contract of the best contract in each category.

#### 4.2.1 Run-Proof Contracts

A run-proof contract solves (15) subject to the additional constraint $x_{2,L} \geq x_1$. The bank can avoid investing in the loan market by setting $a = 1 - \lambda x_1$, in which case $x_{2,L} = x_{2,H}$. Nevertheless, depending on the prevailing loan rate, the bank may optimally invest some funds in the loan market. Lemma 2 implies that when $r$ is small, the optimal deposit contract that solves (15) is run-proof. If $r \geq \underline{R}/\left(1 - p_L\underline{R}\right)$, the solution to (15) violates the run-proof condition, and therefore, the optimal run-proof contract requires that the participation constraint binds with $x_{2,L} = x_1 = a\underline{R}/(1-\lambda)$. In addition, $a$ solves the FOC

$$\left(p_L + \frac{\lambda}{1-\lambda}\right)\underline{R}u'\left(\frac{a\underline{R}}{1-\lambda}\right) - p_H\left(r - \underline{R} + \frac{\lambda r\underline{R}}{1-\lambda}\right)u'_H\left(\frac{r - a\left(r - \underline{R} + \frac{\lambda r\underline{R}}{1-\lambda}\right)}{1-\lambda}\right) = 0$$

The borrower's problem remains the same as in Section 4.1.

Since clear comparative-static results for the contract terms with respect to $\chi$ are unavailable, we do not analytically determine how $\chi$ affects $r$ and the contract. We therefore use a numerical example to illustrate the underlying mechanisms. The parameter values are $\bar{R} = 1.35$, $\underline{R} = 1.14$, $\lambda = 0.5$, $b = 0.01$, $\gamma = 1.5$, $\alpha = 2$, $A = 0.4$, $B = 1$, $\delta = 0.1$, $n_b = 0.15$, and $p_H = 0.997$.

The solid lines in Figure 8 depict the allocations and contract terms under the best run-proof contract. For $\chi < 0.16$, the run-proof constraint is slack, with $x_1 < x_{2,L}$. For $\chi \geq 0.16$, the constraint binds, with $x_1 = x_{2,L}$. As pledgeability increases, greater demand for loans drives up the loan rate and size, although the increase in $r$ is small for $\chi < 0.16$ and not obvious in Figure 8.

For $\chi > 0.16$, unlike Figures 2, 3, and 7, the increase in $r$ induces lower $x_1$, even though depositors' EIS is below one. The decrease in $x_1$ is necessary to keep the no-run constraint satisfied. As future returns get higher, investing in the loan market becomes more attractive. On the one hand, banks tend to give more $x_1$ to smooth consumption; on the other hand, the resulting reduction in safe investment, $a$, lowers $x_{2,L}$. Therefore, the no-run constraint becomes binding for higher $\chi$. As the bank allocates more resources to the loan market, both $x_1$ and $x_{2,L}$ decrease given a binding no-run constraint.

### 4.2.2 Run-Admitting Contract

Next, we formulate a run-admitting contract following Cooper and Ross (1998). A run-admitting contract offers $x_{2,L} < x_1$. Thus, when depositors receive a private signal of State $L$, they will withdraw from the bank at $t = 1$. In that case, the bank then liquidates assets in the safe technology and pays depositors until it runs out of resources. The total measure of depositors served in the default state is $\lambda + a/x_1$. The bank's deposit contract solves

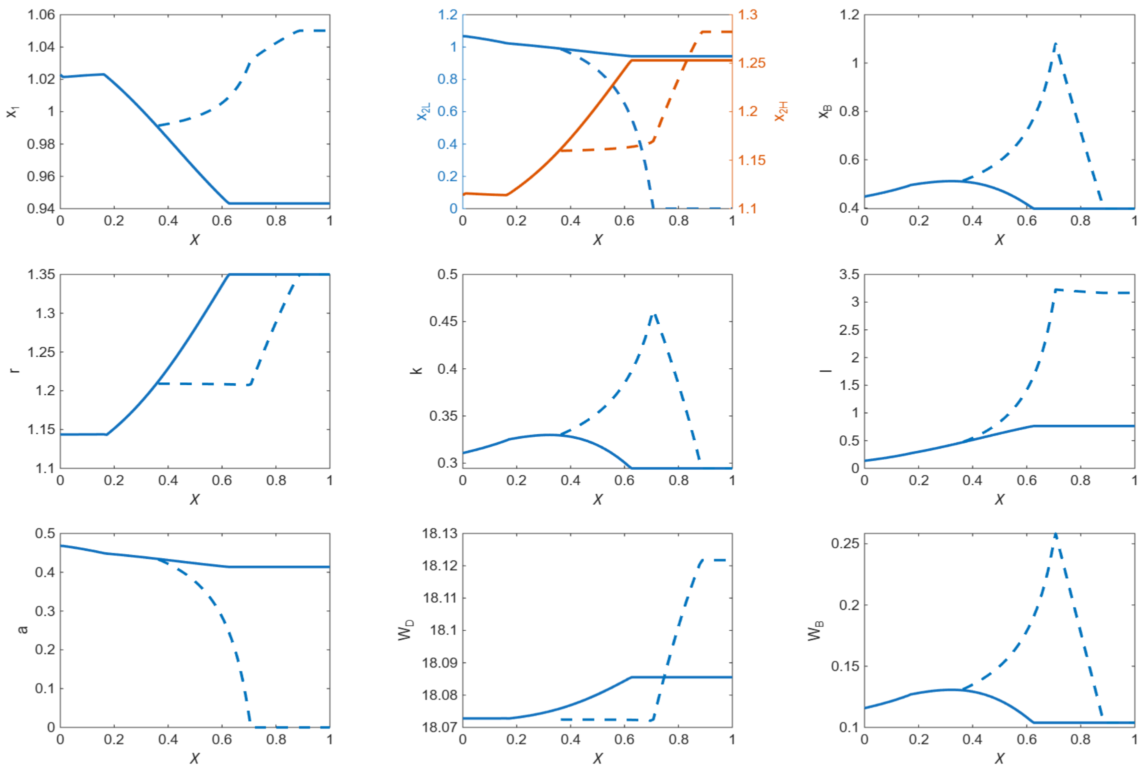
$$\max_{x_1, x_{2,L}, x_{2,H}, a} p_H \left[\lambda u\left(x_1\right) + (1-\lambda) u\left(x_{2,H}\right)\right] + p_L \left(\lambda + \frac{a}{x_1}\right) u\left(x_1\right) \tag{20}$$

$$\text{st } x_1 > \frac{a\underline{R}}{1-\lambda} \tag{21}$$

$$x_{2,H} = \frac{\left(1 - \lambda x_1 - a\right) r + a\underline{R}}{1-\lambda}$$

That is, in State $H$, the depositors do not run, receiving $x_1$ at $t = 1$ and $x_{2,H}$ at $t = 2$. In State $L$, all depositors withdraw at $t = 1$, but only $\lambda + a/x_1$ of them are served. We

Figure 8: Run-proof/Run-admitting Contract




*Note:* The figure shows equilibrium outcomes as collateral pledgeability, $\chi$, varies in the model with stochastic returns and private information. The solid lines represent outcomes under run-proof contracts, and the dashed lines show outcomes under run-admitting contracts. The parameter values are $\bar{R} = 1.35$, $\underline{R} = 1.14$, $\lambda = 0.5$, $b = 0.01$, $\gamma = 1.5$, $\alpha = 2$, $A = 0.4$, $B = 1$, $\delta = 0.1$, $n_b = 0.15$ and $p_H = 0.997$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption in state L $x_{2,L}$, patient depositors' consumption in state H $x_{2,H}$ (orange lines), borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

first solve the problem without imposing constraint (21). We refer to it as the "relaxed problem." If the solution to the relaxed problem violates (21), the constrained solution is characterized by $x_1$ approaching $a\underline{R}/(1-\lambda)$ from above. Throughout the following analysis, we denote this limiting case by $x_1 \to a\underline{R}/(1-\lambda)$.

However, the following lemma shows that a run-admitting contract offering $x_1 \to a\underline{R}/(1-\lambda)$ is strictly dominated by the best run-proof contract. The intuition is as follows: Given $a$, offering $x_1 = a\underline{R}/(1-\lambda)$ lowers early payments and raises late payments by arbitrarily small amounts, leaving the expected utility in State $H$ almost unchanged. In State $L$, however, the resulting contract eliminates depositor's incentive to run and allows all depositors to receive $x_1$ or the equivalent. This yields strictly higher welfare than under a run.

**Lemma 3** *If a run-admitting contract offers $x_1 \to a\underline{R}/(1-\lambda)$, it is strictly dominated by a run-proof contract.*

**Proof.** See Appendix A. ■

Hence, the optimal contract can be determined as follows. If the solution to the relaxed problem violates (21), the optimal contract is run-proof. Otherwise, the bank then compares the resulting run-admitting contract with the best run-proof contract and selects the one that yields higher ex-ante expected utility.

The FOCs for the relaxed run-admitting problem are

$$\begin{aligned} \lambda\left[u'(x_1) - p_H r u'\left(\frac{(1-\lambda x_1 - a)r + a\underline{R}}{1-\lambda}\right)\right] + p_L \frac{a}{x_1}\left[u'(x_1) - \frac{1}{x_1}u(x_1)\right] &= 0 \\ -p_H u'\left(\frac{(1-\lambda x_1 - a)r + a\underline{R}}{1-\lambda}\right)\left(r - \underline{R}\right) + p_L \frac{1}{x_1}u(x_1) &\leq 0 \quad (22) \end{aligned}$$

where condition (22) holds with equality if $a > 0$. The borrower's problem remains the same.

The following proposition shows that when pledgeability is high and the probability of default is small, the optimal run-admitting contract can dominate the optimal run-proof contract.

**Proposition 2** *Suppose $p_L \to 0$. The optimal contract is run-admitting if $\chi > \bar{\chi}$.*

**Proof.** See Appendix A. ■

To illustrate the mechanism, we continue the example in Section 4.2.1 and plot the run-admitting contract terms in Figure 8. The dashed lines show the best run-admitting contract for the relaxed problem. For small $\chi$, a run-admitting contract that solves the relaxed problem is not feasible. This is because low demand for loans drives down $r$, so the bank does not invest in the loan market; and even if it does, the amount is so small that $x_{2,L}$ is close to $x_{2,H}$, and the payments violate (21). In this example, the solution to the relaxed problem satisfies (21) only for $\chi > 0.37$.

Compared with the run-proof contract, loan investment increases more rapidly under a run-admitting contract as $\chi$ rises. The run-admitting contract yields lower welfare than the run-proof contract when $\chi < 0.75$ and higher welfare otherwise. This means that the bank will choose a run-proof contract if $\chi < 0.75$ and a run-admitting contract if $\chi > 0.75$. In other words, bank runs occur only in economies with higher capital pledgeability.

This result seems counterintuitive, but the underlying mechanism is straightforward. A lower $\chi$ depresses the loan rate and encourages more investment in the safe asset, which in turn eliminates runs. In contrast, when pledgeability is high, the higher loan rate makes investment in the risky but more productive technology more attractive. The bank is therefore willing to tolerate a run in the relatively unlikely state $L$ in exchange for higher depositor consumption in state $H$.

# 5 Conclusion

Banking policies and financial indicators are sometimes constructed and interpreted based on conventional wisdom. For example, the Federal Reserve Bank of Chicago's National Financial Conditions Index (NFCI) interprets increases in interest rate spreads as signs of tighter credit conditions. Similarly, during the 2005 debate over the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), which was expected to improve the pledgeability of corporate debt, Posner (2005) argued that the legislation would lower interest rates and benefit borrowers. Although such interpretations are often supported by partial-equilibrium models, our analysis shows that they can be misleading in general equilibrium.

We model loan market friction in an otherwise standard Diamond-Dybvig setup and jointly characterize equilibrium in the loan and deposit markets. In some parameter regions, greater pledgeability expands loan demand and intensifies competition for funds, causing the equilibrium loan rate to rise rather than fall. The interaction between deposit supply and loan demand can also generate nonmonotonic responses in loan size, collateral production, deposit payments, and other contract terms. Thus, once general-equilibrium interactions are taken into account, contract terms need not vary monotonically with the underlying financial friction and may even display counter-intuitive patterns.

Because our framework incorporates both deposit and loan markets, the equilibrium intermediary contracts reveal how these interactions affect intermediaries' asset composition, loan risk, and risk sharing among depositors. We also extend the model to study financial stability in the presence of stochastic loan returns and borrower default. Low pledgeability depresses the loan rate and encourages investment in safe assets, making a run-proof contract optimal. By contrast, high pledgeability can make a run-admitting contract optimal. Greater pledgeability can therefore increase financial fragility by inducing banks to accept greater exposure to run risk.

In future work, we plan to introduce dynamics and examine stochastic variation in pledgeability at both growth and business-cycle frequencies. We also aim to study how economy-wide changes in credit conditions affect heterogeneous agents and to evaluate the model's predictions empirically.

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# Appendix

## A Proofs

**Proof of Proposition 1**: First, we show the uniqueness of the equilibrium. Rewrite the slope of the loan supply curve in the Medium Region, given by (6), as

$$\frac{d\ell^s}{dr} = \frac{u'(x_2) + x_2 u''(x_2)}{-\frac{u''(x_1)}{\lambda} - \frac{r^2 u''(x_2)}{1-\lambda}} > \frac{x_2 u''(x_2)}{-\frac{u''(x_1)}{\lambda} - \frac{r^2 u''(x_2)}{1-\lambda}} > \frac{x_2 u''(x_2)}{-\frac{r^2 u''(x_2)}{1-\lambda}} = -\frac{\ell}{r}$$

The first inequality follows from the fact that $u' > 0 > u''$, the second is by the fact that $-u_1''/\lambda$ is positive and $u_2''$ is negative, and the last equality holds because $(1-\lambda)\, x_2 = \ell r$ in the Medium Region. This implies that if $\ell^s$ is decreasing, the absolute value of its slope is less than $\ell/r$.

Similarly, rewrite the slope of the loan demand curve, given by (12) as

$$\begin{aligned}
\frac{d\ell^d}{dr} &= \frac{\ell}{r} \frac{-\frac{\left(\bar{R}-1+\chi\right)^2}{\ell} c' - r\left(r - \bar{R} + 1 - \chi\right) c'' + r^2 (1-\chi)\, v''}{\left(r - \bar{R} + 1 - \chi\right)^2 c'' - r^2 (1-\chi)^2 v''} \\
&< \frac{\ell}{r} \frac{-r\left(r - \bar{R} + 1 - \chi\right) c'' + r^2 (1-\chi)\, v''}{\left(r - \bar{R} + 1 - \chi\right)^2 c'' - r^2 (1-\chi)^2 v''} \\
&= \frac{\ell}{r} \frac{-r\left(r - \bar{R} + 1 - \chi\right) c'' + r^2 (1-\chi)\, v''}{r\left(r - \bar{R} + 1 - \chi\right) c'' - \left(\bar{R} - 1 + \chi\right)\left(r - \bar{R} + 1 - \chi\right) c'' - r^2 (1-\chi)^2 v''} \\
&< -\frac{\ell}{r}
\end{aligned}$$

The first inequality follows from the fact that $c', c'' > 0 > v''$ and the second inequality follows from the fact that $c'' > 0$. This implies that the demand curve is downward-sloping and the absolute value of the slope is bigger than $\ell/r$.

Hence, if the demand and supply curves intersect in the Medium Region, the demand curve must cut through the supply curve from above. Multiple intersections would require at least one crossing in the opposite direction, contradicting the slope ordering established above. A contradiction. Therefore, the equilibrium in the Medium

Region is unique.

If there is an equilibrium in the Medium Region, then $\ell^d > \ell^s$ at $r = \underline{R}$, and $\ell^d < \ell^s$ at $r = \bar{R}$, which means the equilibrium is unique. If $\ell^d$ and $\ell^s$ intersect at $r = \underline{R}$, it must be that $\ell^d \leq 1 - \lambda \hat{x}_1$. To get another equilibrium in the Medium or High Region, $\ell^d$ has to cross $\ell^s$ from below. A contradiction. Similar argument applies to the case where $\ell^d$ and $\ell^s$ intersect at $r = \bar{R}$. Therefore, the equilibrium is unique.

Next, we show there are only two cutoffs separating the three regions. It is straightforward to show that if $\chi \geq \bar{\chi}$, the equilibrium lies in the High Region and $\eta = 0$. Now consider $\chi < \bar{\chi}$ with $\eta > 0$. In the Low Region, $a > 0$ and $r = \underline{R}$. By (5), $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$. By (7)-(9), the demand for $\ell$ is characterized by

$$-c'\left(\frac{1-\chi+\underline{R}-\bar{R}}{\bar{R}-1+\chi}\ell\right)\left(1-\chi+\underline{R}-\bar{R}\right)+(1-\chi)\,\underline{R}v'\left(\frac{1-\chi}{\bar{R}-1+\chi}\ell\right)=0 \tag{23}$$

Take derivative wrt $\chi$ to get

$$\frac{d\ell}{d\chi}=-\frac{c'\left(\bar{R}-\underline{R}\right)\left(\bar{R}-1+\chi\right)^2+\ell\underline{R}\,(1-\chi)\left[c''\left(1-\chi+\underline{R}-\bar{R}\right)-v''\underline{R}\bar{R}\right]}{(1-\chi)\left(\bar{R}-1+\chi\right)\left[\underline{R}^2(1-\chi)^2v''-\left(1-\chi+\underline{R}-\bar{R}\right)^2c''\right]}>0$$

By (13) and since $x_1$ is constant, $da/d\chi < 0$. Thus, the cutoff $\underline{\chi}$ between the Medium and Low Regions is unique. ■

**Proof of Lemma 1**: If $a$ hits the corner of $a = 1 - \lambda x_1$, then $x_{2,L} = x_{2,H}$, and the FOC wrt $a$ results in $\underline{R} - p_H r \geq 0$. Because of the assumption of $p_H\bar{R} > \underline{R}$, we have $r < \bar{R}$. There is no supply of loan since all assets are in the safe technology. The borrower's FOCs implies that $\ell^d = 0$ if and only if $r = \bar{R}$. Therefore, the loan market does not clear. ■

**Proof of Lemma 2**: First, we show if $r < \underline{R}/\left(1 - p_L\underline{R}\right)$, the solution has $x_{2,L} > x_1$. Note $a$ must be positive to get $x_{2,L}$>0. Then combine (17)-(18) to get

$$p_L u'\left(x_{2,L}\right)\underline{R}r - u'\left(x_1\right)\left(r - \underline{R}\right) = 0 \tag{24}$$

As $r < \underline{R}/\left(1 - p_L\underline{R}\right)$, $u'\left(x_{2,L}\right) < u'\left(x_1\right)$. By the concavity of $u$, $x_{2,L} > x_1$. Next, we show if $x_{2,L} > x_1$, then $r < \underline{R}/\left(1 - p_L\underline{R}\right)$. Note that since consumption is non-negative, $x_{2,L}$ must be positive, which implies $a$ is positive. Again use the FOCs to get (24). By the concavity of $u$, $x_{2,L} > x_1$ results in $r < \underline{R}/\left(1 - p_L\underline{R}\right)$. ■

**Proof of Lemma 3**: First, we show $a/x_1 < 1-\lambda$. Otherwise, all depositors are paid $x_1$ in period 1 and the expected utility is $p_H\left[\lambda u\left(x_1\right) + \left(1-\lambda\right)u\left(x_{2,H}\right)\right] + p_L u\left(x_1\right)$. Lowering $a$ improves depositor's welfare as

$$\frac{dW^{ra}}{da} = -p_H u_H' \left(r - \underline{R}\right) < 0.$$

Next, consider a run-proof contract offering $\tilde{x}_1 = x_1 - \varepsilon = a\underline{R}/(1-\lambda) = x_{2,L}$, where $\varepsilon > 0$. Let $\tilde{x}_{2,H}$ represent the corresponding late payment in State $H$. Denote the expected utility under the run-admitting contract by $W^{ra}$, and under the run-proof contract by $W^{rp}$. The difference is

$$\begin{aligned} W^{ra} - W^{rp} &= \lambda\left[u\left(x_1\right) - u\left(\tilde{x}_1\right)\right] + p_H\left(1-\lambda\right)\left[u\left(x_{2,H}\right) - u\left(\tilde{x}_{2,H}\right)\right] + p_L\left[\frac{a}{x_1}u\left(x_1\right) - \left(1-\lambda\right)u\left(x_{2,L}\right)\right] \\ &\rightarrow p_L\left[a - \left(1-\lambda\right)x_1\right]\frac{u\left(x_1\right)}{x_1} < 0 \end{aligned}$$

■

**Proof of Proposition 2**: We first show that the solution to (15) has $x_1 > x_{2,L}$ if $\chi > \bar{\chi}$. If $a = 0$, it is obvious that $x_1 > 0 = x_{2,L}$. If $a > 0$, by (17) and (18), $\left(r - \underline{R}\right)u'\left(x_1\right) = p_L r u'\left(x_{2,L}\right)$. As $\chi > \bar{\chi}$, $r = \bar{R}$. By the assumption that $p_H\bar{R} > \underline{R}$, we have $p_L\bar{R} < \bar{R} - \underline{R}$ and thus $x_1 > x_{2,L}$ by Lemma 2. Let $p_L \rightarrow 0$. Depositor's expected utility converges to $\lambda u\left(x_1\right) + \left(1-\lambda\right)u\left(x_{2,H}\right)$, which is the same as (15). The solution to (15) also solves (20). Since the run-proof contracts are constrained to have $x_1 \leq x_{2,L}$, they cannot attain this allocation. Therefore, the optimal contract is run-admitting. ■

# B Market Power

To test the robustness of the non-monotonicity results, we examine the market response to changes in pledgeability under alternative loan market structures. Here, we use search frictions as a way to incorporate market power into the loan market.

## B.1 Nash Bargaining

First, consider bilateral matches in the loan market. Banks and borrowers meet bilaterally to decide the terms of the loan contract. If they do not agree on the terms of trade, there is no trade and the payoffs are $\hat{W}_D$ and $\hat{W}_B$, respectively. Otherwise, they jointly determine the contract terms according to the generalized Nash bargaining solution. Let $\theta$ be the bargaining, or market, power of the bank. The generalized Nash problem is

$$\max_{x_1,x_2,x_B,r,k,a} \left[\lambda u\left(x_1\right) + (1-\lambda)\, u\left(x_2\right) - \hat{W}_D\right]^{\theta} \left[-c\left(k\right) + v\left(x_B\right) - \hat{W}_B\right]^{1-\theta} \tag{25}$$

$$\text{st (3), (4)}$$

$$x_B = (1-\lambda x_1 - a)\left(\bar{R} - r\right) + k\bar{R} \tag{26}$$

$$x_B \geq (1-\chi)(1-\lambda x_1 - a + k) \tag{27}$$

The first-order conditions with respect to $(x_1, r, k, a)$ simplify to

$$u'\left(x_1\right)\left[c'\left(k\right) - (1-\chi)\, v'\left(x_B\right)\right] - \left(\bar{R} - 1 + \chi\right) u'\left(x_2\right) c'\left(k\right) = 0 \tag{28}$$

$$a\left[-u'\left(x_1\right) + \underline{R} u'\left(x_2\right)\right] = 0 \tag{29}$$

$$\theta u'\left(x_1\right) S_B - (1-\theta)\, c'\left(k\right) S_D = 0 \tag{30}$$

$$(1-\chi)\,\eta - \frac{(1-\theta)\, S_D c'\left(k\right)}{u'\left(x_1\right)}\left[\bar{R} u'\left(x_2\right) - u'\left(x_1\right)\right] = 0 \tag{31}$$

where the bank's trade surplus is $S_D \equiv \lambda u\left(x_1\right) + (1-\lambda)\, u\left(x_2\right) - \hat{W}_D$, the borrower's trade surplus is $S_B \equiv -c\left(k\right) + v\left(x_B\right) - \hat{W}_B$, and $\eta$ is the Lagrangian multiplier associated with (27). As in the competitive market, the equilibrium falls into one of three regions depending on $\chi$. With a slight abuse of notation, we again use $\underline{\chi}$ and $\bar{\chi}$ to denote the cutoffs between the regions.

**High-pledgeability Region**: With $\eta = 0$ and $a = 0$, this region is characterized by $u'(x_1)/u'(x_2) = c'(k)/v'(x_B) = \bar{R}$, which implies that only borrower's higher-return technology is used and that the allocation is efficient. However, as $\theta \in (0, 1)$, the bank does not get all surplus. Thus, $r < \bar{R}$. Denote the solution by $\left(\tilde{x}_1, \tilde{k}, \tilde{r}\right)$. The contract is in the High Region if and only if $\chi \geq \bar{\chi} \equiv 1 - \bar{R} + \tilde{r}(1 - \lambda \tilde{x}_1)/\left(1 - \lambda \tilde{x}_1 + \tilde{k}\right)$.

**Medium-pledgeability Region**: With $\eta > 0$ and $a = 0$, the contract is in the Medium Region if and only if $\underline{\chi} \leq \chi < \bar{\chi}$. Here, $\underline{R} \leq u'(x_1)/u'(x_2) < \bar{R}$, which implies the bank also undertakes the cost of twisting the investment in $k$ by reducing the loan size and/or the rate. On the borrower's side, $c'(k) > \bar{R}v'(x_B)$, which implies that the marginal investment in $k$ relaxes the borrowing constraint.

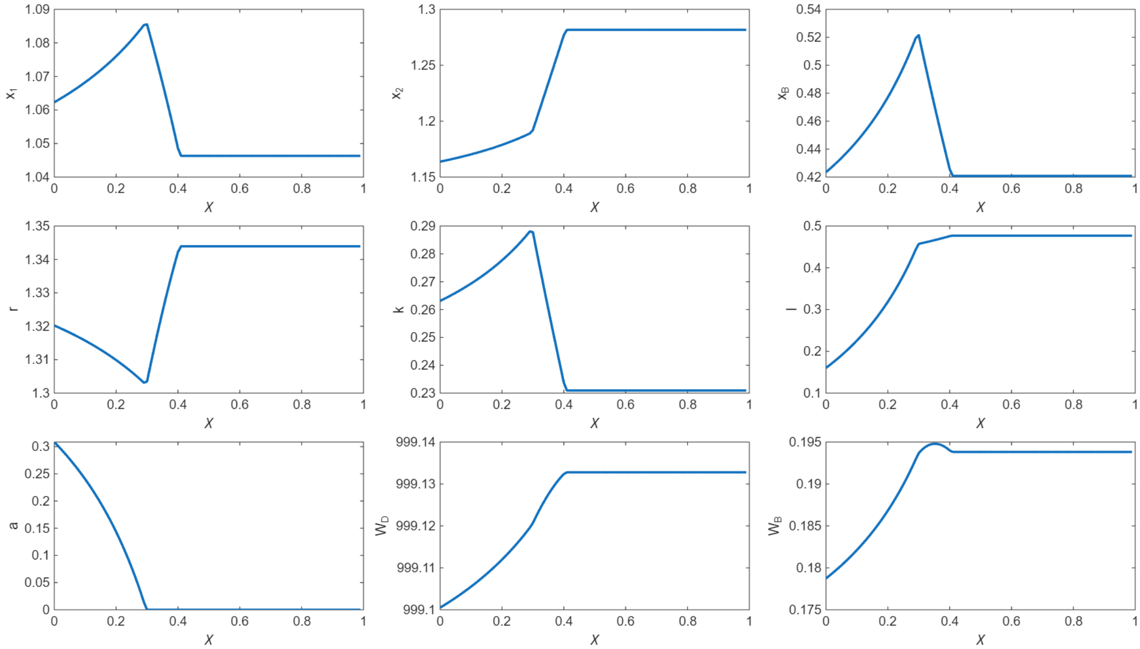
**Low-pledgeability Region**: With $\eta > 0$ and $a > 0$, the loan contract is in this Region if and only if $\chi < \underline{\chi}$. Again, $c'(k) > \bar{R}v'(x_B)$. On the bank's side, $u'(x_1) = \underline{R}u'(x_2)$ as the marginal investment of the bank yields $\underline{R}$.

Loan terms again exhibit non-monotonic patterns, but for a different reason. In a bilateral negotiation, both parties fully internalize all terms of the contract, including $r$, to adapt to changes in $\chi$. The marginal value of each term is different when the repayment constraint binds. Consequently, the loan terms may not change in the same magnitude or even in the same direction with $\chi$. Furthermore, one's surplus does not necessarily increase with the expansion of the Nash bargaining set (Kalai, 1977).

There is one clear comparative static. The quantity of direct investment is inversely related to pledgeability in the Low Region. Because the borrower's project return dominates the risk-free rate, the safe asset is the last tool that the bank uses to cope with the deteriorating pledgeability.

We continue the numerical analysis in Figures 2 and 3 set $\theta = 0.5$. Figures B.1 and B.2 present the equilibrium contracts.

Figure B.1: Nash Bargaining ($\gamma = 2, \delta = 0.5$)

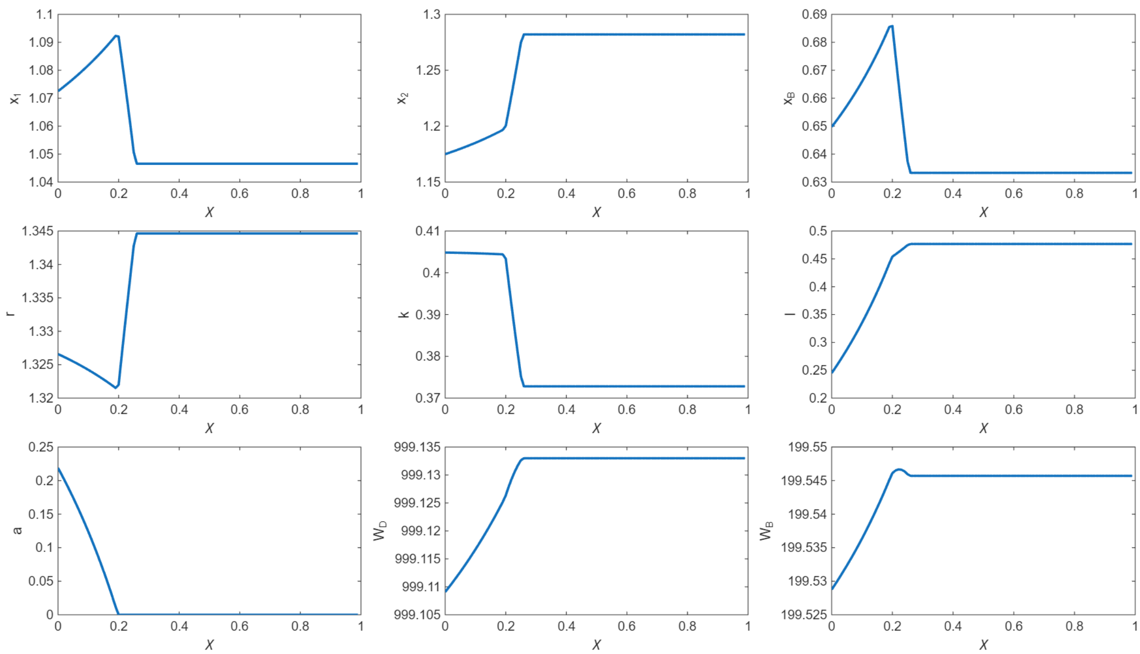



*Note:* The figure shows Nash bargaining outcomes as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 2$ and the borrower's to $\delta = 0.5$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

## B.2 Competitive Search

We can endogenize the bargaining power by considering competitive search. Suppose a bank can open a loan market by posting the terms of borrowing $(x_1, x_2, x_B, r, k, \ell)$. Borrowers observe the terms and pay an entry cost, denoted by $\phi$, choosing to go to a specific market. In a market, banks and borrowers are matched according to the matching function $M(n_D, n_B)$, where $n_D$ and $n_B$ are the measures of banks and borrowers, respectively. Assume $M$ is strictly increasing, strictly concave, and homogeneous of degree 1 in both arguments. Let $\tau = n_B/n_D$ be the market tightness. The probability that a bank meets a borrower is $\sigma(\tau) \equiv M(1, \tau)$ and the probability that a borrower

Figure B.2: Nash Bargaining ($\gamma = 2, \delta = 2$)




*Note:* The figure shows Nash bargaining outcomes as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 2$ and the borrower's to $\delta = 2$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor welfare $W_D$, and borrower welfare $W_B$.

meets a bank is $\sigma(\tau)/\tau$. Banks post terms of trade to solve the following problem:

$$\max_{x_1,x_2,x_B,r,k,a,\tau} \sigma(\tau)\left[\lambda u(x_1) + (1-\lambda)u(x_2)\right] + \left[1-\sigma(\tau)\right]\hat{W}_D \tag{32}$$

$$\text{st (3), (4), (26), (27)}$$

$$\frac{\sigma(\tau)}{\tau}\left\{-c(k) + v\left[(1-\lambda x_1 - a)\left(\bar{R}-r\right) + k\bar{R}\right]\right\} + \left[1-\frac{\sigma(\tau)}{\tau}\right]\hat{W}_B - \phi = \hat{W}_B \tag{33}$$

The LHS of (33) is the expected utility of a borrower if he enters the market, and the RHS is his payoff if he does not.

The first-order conditions are given by (28), (29) and

$$
\begin{aligned}
[1-\varepsilon(\tau)]\, u'(x_1)\, S_B - \varepsilon(\tau)\, c'(k)\, S_D &= 0 \qquad (34)\\
\eta - \frac{\sigma(\tau)\, u_2'(x_2)}{c'(k) - (1-\chi)v'(x_B)}\left[c'(k) - \bar{R}v'(x_B)\right] &= 0 \qquad (35)
\end{aligned}
$$

where $\varepsilon(\tau) = \sigma'(\tau)\,\tau/\sigma(\tau)$ is the elasticity of the matching function and $\eta$ is the Lagrangian multiplier associated with (27). With competitive search, the bargaining power is endogenized by the elasticity of the matching function. Note that if the matching elasticity is constant, the solution is the same as under Nash bargaining.

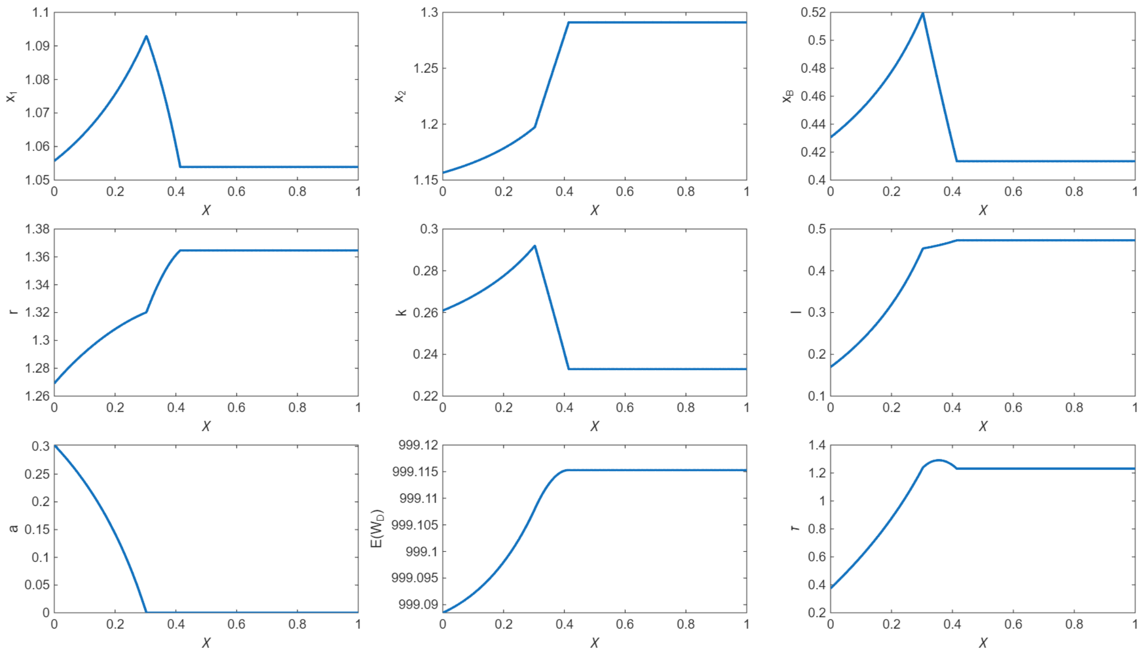
The equilibrium again falls into one of three regions associated with pledgeability. The comparative statics depend on the matching function and are generally ambiguous. The numerical analysis uses the same parameters as in Figures 2 and 3. Let $M(n_D, n_B) = n_D n_B/(n_D + n_B)$ and $\phi = 0.01$.

Figure B.3 and B.4 plot the equilibrium contract terms, which are qualitatively similar to what we observed under Nash bargaining. Here, the loan rate may rise with pledgeability in the entire range (Figure B.3). The market gets tighter when $\chi$ increases from 0, but gets looser in the Medium Region and stays constant in the High Region. The ex-ante welfare of the borrowers is $\phi$ by the free entry condition, while the welfare of banks strictly increases for $\chi \leq \bar{\chi}$, despite the non-monotone matching probability. Using parameters in Figures 4 and 5 produces the same patterns.

Suppose the entry cost rises. If $\varepsilon' < 0$, we can show that $\underline{\chi}$ and $\bar{\chi}$ strictly decrease in $\phi$. A high entry cost discourages borrowers from entering the loan market, and the market becomes less tight. So banks offer more favorable terms to borrowers, and borrowers are more willing to pay off loans rather than default. Hence, the repayment constraint is relaxed, the High-region expands, and the Low-Region contracts.

Table B.1 presents the comparative statics. As $\phi$ increases, from the perspective of credit market indicators, credit conditions appear to improve, reflected in lower interest rates and reduced collateral requirements. However, aggregate welfare declines as depositors are worse off by offering more favorable terms to borrowers.

Figure B.3: Competitive Search ($\gamma = 2, \delta = 0.5$)




*Note:* The figure shows the equilibrium outcome with competitive search loan market as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 2$ and the borrower's to $\delta = 0.5$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor's ex-ante welfare $E(W_D)$, and market tightness $\tau$.

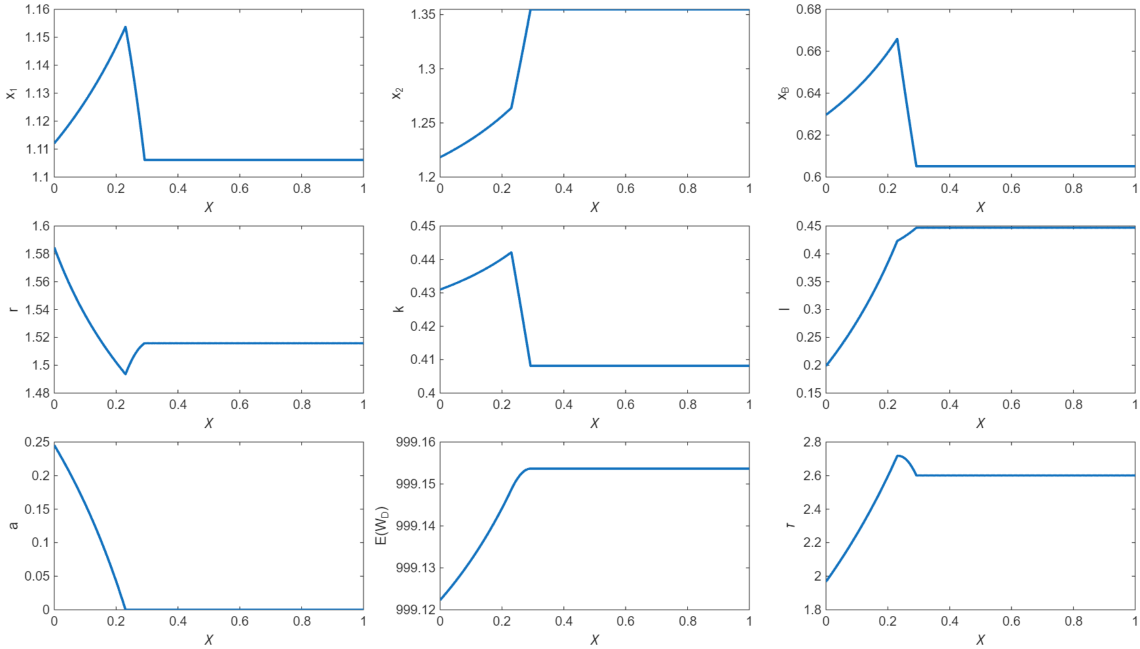
Table B.1: Competitive Search: Comparative Statics of Entry Cost ($\varepsilon' < 0$)

| | $dx_1/d\phi$ | $dx_2/d\phi$ | $dx_B/d\phi$ | $dk/d\phi$ | $dr/d\phi$ | $d\ell/d\phi$ | $da/d\phi$ |
|---|---|---|---|---|---|---|---|
| High | − | − | + | − | − | + | N.A. |
| Medium | ± | ± | ± | − | − | ± | N.A. |
| Low | − | − | + | − | − | + | − |

*Note:* The table reports comparative statics for the equilibrium in the competitive-search loan market as the borrowers' entry cost, $\phi$, varies, given that the elasticity of the matching function's derivative satisfies $\varepsilon' < 0$. The three rows correspond to the High-, Medium-, and Low-pledgeability Regions. The signs +, −, and 0 indicate whether the variable increases, decreases, or remains unchanged as $\chi$ increases. The symbol ± indicates that the sign is ambiguous and depends on parameter values, while N.A. denotes that the variable is not defined in that region.

We use the same parameter values as in Figure 2, fix $\chi = 0.3$, and vary the entry cost $\phi$. Figure B.5 plots the resulting contract terms. Notice that with a higher $\phi$, impatient depositors are worse off than patient depositors, implying that partial insurance is also

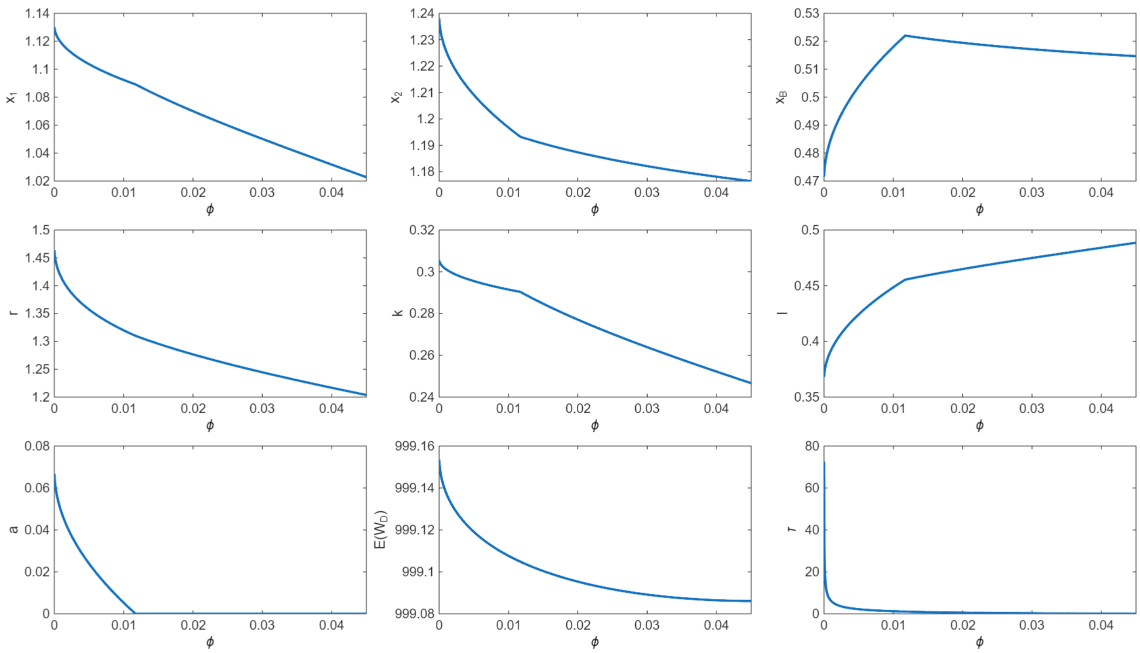
Figure B.4: Competitive Search ($\gamma = 2, \delta = 2$)



*Note:* The figure shows the equilibrium outcome with competitive search loan market as collateral pledgeability $\chi$ varies, with the depositor's EIS parameter set to $\gamma = 2$ and the borrower's to $\delta = 2$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor's ex-ante welfare $E(W_D)$, and market tightness $\tau$.

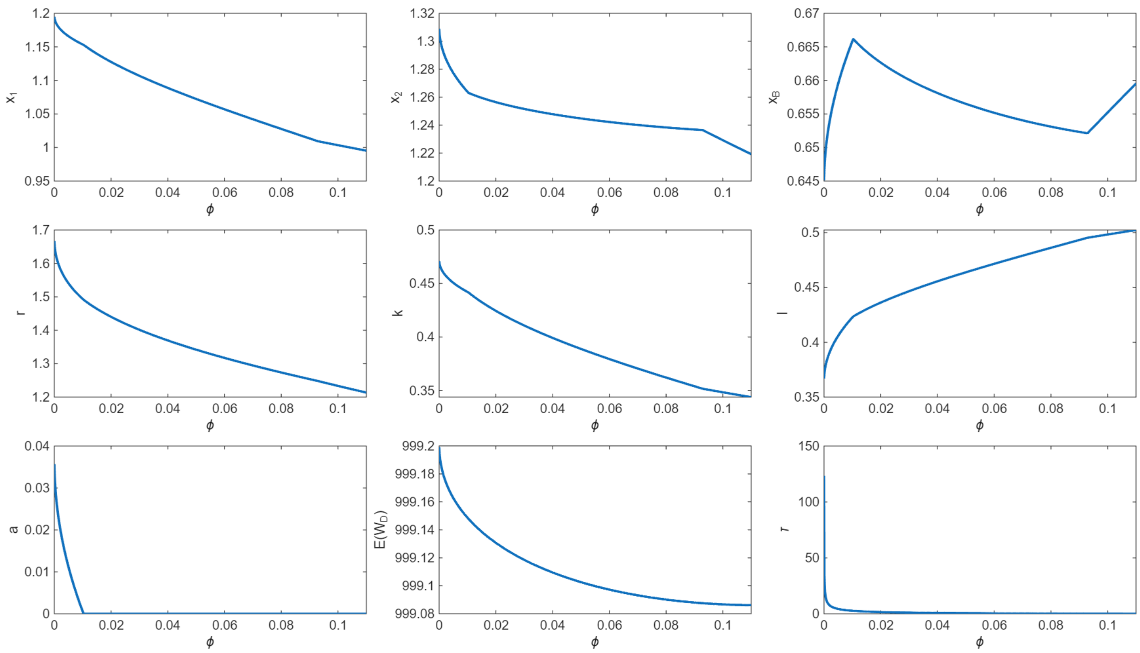
weaker. Using parameters in Figure 3 and fixing $\chi = 0.23$ produces similar results, as shown in Figure B.6.

Figure B.5: Competitive Search: Varying Entry Costs ($\gamma = 2, \delta = 0.5$)



*Note:* The figure shows the equilibrium outcome with competitive search loan market as borrowers' entry cost $\phi$ varies, with the depositor's EIS parameter set to $\gamma = 2$ and the borrower's EIS $\delta = 0.5$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor's ex-ante welfare $E(W_D)$, and market tightness $\tau$.

Figure B.6: Competitive Search: Varying Entry Costs ($\gamma = 2, \delta = 2$)




*Note:* The figure shows the equilibrium outcome with competitive search loan market as borrowers' entry cost $\phi$ varies, with the depositor's EIS parameter set to $\gamma = 2$ and the borrower's to $\delta = 2$. The panels show impatient depositors' consumption $x_1$, patient depositors' consumption $x_2$, borrower consumption $x_B$, the loan rate $r$, the borrower's capital production $k$, loan size $\ell$, the bank's investment in the safe technology $a$, depositor's ex-ante welfare $E(W_D)$, and market tightness $\tau$.