Optimal Intermediary Contracts

Nabi Arjmandi* Chao Gu† Joseph H. Haslag‡ Yajie Wang§

December 8, 2024

Abstract

We study an economy where financial intermediaries offer deposit contracts to partially insure lenders against idiosyncratic risks and extend collateralized loans to borrowers with limited commitment. Our analysis highlights the critical role of pledgeability in shaping optimal intermediary contracts. Specifically, we identify a nonmonotonic relationship between contract terms and pledgeability across different market structures, including perfect competition, bilateral trade, and competitive search. Our theoretical results can explain the puzzling response of corporate loans following the passage of the Bankruptcy Abuse Prevention and Consumer Protection Act. In addition, we investigate the impact of unexpected shocks on financial stability and extend the model to explore how pledgeability influences welfare under the Glass-Steagall Act.

JEL Codes: E40, E61, E62, H21.

Keywords: optimal deposit contract, optimal loan contract, pledgeability, bank risk, Bankruptcy Abuse Prevention and Consumer Protection Act, Glass-Steagall Act.

*Department of Economics, University of Missouri-Columbia, USA. Email: arjmaindim@mail.missouri.edu.
†Department of Economics, University of Missouri-Columbia, USA. Email: guc@missouri.edu.
‡Department of Economics, University of Missouri-Columbia, USA. Email: haslagj@missouri.edu.
§Department of Economics, University of Missouri-Columbia, USA. Email: yajie.wang@missouri.edu.
The authors wish to thank Phil Dybvig, Peter Haslag, Ping Wang, Cathy Zhang, and conference participants at the 2023 Midwest Macro meetings, the 2024 Costa Rica Search and Matching Conference, and the 2024 Summer Workshop on Money, Banking, Payments, and Finance for helpful comments and suggestions.

1 Introduction

Financial intermediaries engage in transactions with both borrowers and lenders. Since loan risk affects both the demand and supply sides of the loan market, the structure of intermediary contracts is highly dependent on the level of risk undertaken. A common strategy to mitigate loan risk is the use of collateralized loans. Our goal is to examine the relationship between asset pledgeability and the terms of loan and deposit contracts.

The conventional wisdom regarding this relationship is based on a partial-equilibrium approach. By holding everything else constant, one can predict how changes in loan risk affect a specific term in the contract. This conventional reasoning is reflected in both legislation and finance. Here are two examples.

On the legislative front, the United States passed the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) in 2005. While much of the research has centered on its impact on consumers, such as stricter qualifications for Chapter 7's "fresh start" provision, the Act also introduced significant changes affecting firms.[1] For corporate bankruptcies, particularly under Chapter 11, BAPCPA enhanced creditor protections and improved the pledgeability of corporate bonds. As BAPCPA was being debated, Posner asserted that the law would reduce interest rates and make borrowers better off.[2] The logic is as follows: by enacting BAPCPA, a larger fraction of the loan is pledged for debt repayment. With an increase in collateral, loan risk is reckoned as lower and hence, the loan rate is lower because borrowers require less risk compensation.[3]

However, as shown in Section 2, our empirical findings reveal that the response of corporate loans to BAPCPA is different from the predictions of this canonical logic. First,

[1]For the impact on consumers, see Gross et al. (2021) for a detailed discussion.

[2]See Posner (2005).

[3]See Gupta et al. (2022) for a description of changes in real estate values backing loans. Note that the legal change represents an ex-ante change in the collateral value. With a different set of rules, new loans are subject to a different level of pledged collateral at the time the loan is made. In contrast, changing economic conditions are a kind of ex-post value change. After the loan was made, the collateral value unexpectedly changed.

we find that the implementation of BAPCPA did not lead to a reduction in corporate loan spreads, either on impact or over time. Instead, the pre-BAPCPA downward trend in loan spreads came to a halt after the law was enacted. Second, BAPCPA led to a noticeable reduction in corporate loan sizes upon its enactment, and the previously increasing trend in loan size experienced a significant slowdown after the law's passage. These two puzzling responses, taken together, indicate that conventional reasoning alone is insufficient to fully capture the impact of improved pledgeability on loan contracts.

The second example bears on the measurement of aggregate credit conditions. The Federal Reserve Bank of Chicago has constructed a measure of credit market conditions that includes multiple interest rate spreads and credit quantities. By holding everything else constant, credit conditions worsen as interest rate spreads (including loan rates) increase. If assets are less pledgeable, conventional wisdom is that loan rates monotonically increase as lenders require compensation for the increased risk associated with reduced pledgeability. Through this mechanism, reduced pledgeability leads to a monotonic worsening of credit conditions.

However, it is more likely that all contract terms are adjusted simultaneously in response to changing conditions. How confident can we be that the spread between the loan rate and a risk-free rate accurately reflects changes in risk? For instance, consider a scenario where risk increases but collateral increases sufficiently to cause loan rates to decline. Based solely on the spread between the loan rate and the risk-free rate, the economy would appear to signal a reduction in overall loan risk, even though the underlying risk has actually increased.

In this paper, we construct a model in which contracts are determined in a general equilibrium framework to evaluate the robustness of conventional wisdom regarding the relationship between pledgeability and contract terms. Specifically, we are interested in whether contract terms are monotonically related to pledgeability. To do this, we extend the canonical Diamond-Dybvig model in several distinct ways.[4]

First, we introduce borrowers with limited commitment to loan repayment. To mitigate this friction, borrowers can pledge a fraction of their assets toward debt repayment. The pledgeability of the asset, treated as an exogenous and known parameter, captures the repayment risk associated with the loan. Second, we expand the bank's portfolio options by including a low-return, safe, long-term asset, to allow for a diversification channel. Although the return on loans exceeds that of the safe asset, increasing loan risk or decreasing pledgeability tightens the repayment constraint, eventually forcing banks to allocate some resources to the safe asset.[5]

We consider a competitive loan market as a baseline. The equilibrium contract falls into one of three regions depending on the pledgeability. The High-pledgeability Region corresponds to equilibrium in which the borrower's repayment constraint is non-binding. (The designation applies because the repayment constraint is non-binding for the highest set of pledgeability values.) In the Medium-pledgeability Region, the repayment constraint binds, but the bank chooses to not invest in the low-return safe asset. In the Low-pledgeability Region, the borrower's repayment constraint binds, and banks invest some in the low-return safe asset. Contrary to conventional wisdom, loan terms, including collateral requirement, loan size and rate, and deposit rates, are not monotonic in pledgeability. In particular, the loan rate and collateral size can *increase*, loan size can *decrease*, and borrowers can be worse off when pledgeability improves.

This counterintuitive result is driven by the elasticity of supply and demand. While higher pledgeability relaxes the repayment constraint and stimulates loan demand, competition for loans raises loan rates. If the lender's intertemporal elasticity of substi-

[4]Our work builds on a long tradition of models analyzing the role of banks and secondary markets in providing partial insurance, including Bryant (1980), Jacklin (1987), Bhattacharya and Gale (1987), Hellwig (1994), Diamond (1997), Holmstrom and Tirole (1998), Von Thadden (1999), Allen and Gale (2003), Caballero and Krishnamurthy (2004), and Farhi et al. (2009).

[5]The risk-free asset could be interpreted as a direct investment project undertaken by the bank. This model structure builds on a rich body of literature that analyzes optimal loan contracts under various information frictions. For a summary of the literature on collateral in loan contracts, see, for example, Williamson (1987), Besanko and Thakor (1987), Bernanke and Gertler (1990), Berger and Udell (1990), and Dowd (1992).

tution is sufficiently low, the equilibrium loan volume may decrease. Similarly, if the borrower's intertemporal elasticity of substitution is low, they may offset reduced loan sizes by producing more assets and the collateral-to-loan ratio increases.

To check the robustness of our nonmonotonicity results, we extend the model to explore different market structures, including bilateral trades in an OTC-like market and competitive search. In both settings, loan and deposit terms remain nonmonotonic in pledgeability, and the patterns are sensitive to the market structure. The underlying mechanism differs from that in the baseline: agents can adjust multiple elements of the contracts, which are at different margins when the repayment constraint binds. Thus, changes in pledgeability do not necessarily cause all terms to move uniformly in magnitude or even direction. Despite these complexities, a common feature emerges across market structures: investment in the low-return, safe asset falls with pledgeability in the Low-pledgeability Region, and becomes zero when it crosses into the Medium Region. In other words, using the low-return technology is the last resort under the worst credit conditions because adjusting contract terms does not change the production frontier, while investing in low-return technology contracts it.

We further extend the model to ask three additional questions related to fundamental banking outcomes: (i) will lower entry costs improve welfare? (ii) is financial stability uniformly worsened with declining pledgeability? and (iii) are there conditions under which the Glass-Steagall Act (hereafter the Act) is welfare-improving in this environment? For the first question, we find that with lower entry costs, more borrowers enter the market, receiving less favorable terms, while depositors benefit.

To examine financial stability, we consider an unanticipated reduction in the return on borrowers' projects. Under this unexpected shock, borrowers may default *ex post*, potentially triggering bank runs. In the High- or Medium-Pledgeability Regions, banks invest only in loans. If borrowers default, the bank lacks resources to pay depositors, resulting in bank runs. In the Low-Pledgeability Region, however, banks diversify by investing in both loans and safe assets. These safe assets can be used to pay late depositors even if loan repayments fail. The upshot is that bank runs are less likely in the Low-pledgeability Region than in the Medium or even High-pledgeability Region. In other words, a deterioration in pledgeability can, paradoxically, make the financial industry more stable compared to economies with more pledgeable assets.

For the third question, we modify the model to include stochastic returns on banks' alternative investments. Deposit insurance is provided to aggregate risks, but it also engenders moral hazard. Without the Act, banks are better able to diversify their portfolios, but supporting the insurance program diverts resources from productive activities and encourages excessive risk taking. Interestingly, the Act can improve depositor's welfare in cases of low or high pledgeability, but not when pledgeability is in the medium range. The overall impact of the Act is ambiguous, depending on the level of pledgeability, the riskiness of the investment, and frictions in alternative asset markets.

Related literature

By explicitly linking deposit contracts to loan contracts, our model contributes to the theoretical literature. Following Diamond and Dybvig (1983), a vast body of work explores the role of banks in providing partial insurance to consumers facing idiosyncratic liquidity shocks (see footnote 4). Similarly, a substantial literature examines optimal loan contracts and credit rationing (e.g., Kiyotaki and Moore, 1997; Holmström and Tirole, 1998; Jermann and Quadrini, 2012, among others). Our paper bridges these two strands of research.

Our model shares some features with Antinolfi and Prasad (2008). In their framework, collateral serves as a means of liquidity provision. By pooling collateral, banks allocate resources more efficiently than individuals. The debt repayment constraint may not bind for the bank, even though it would bind for individuals. Another paper closely related to ours is Rocheteau et al. (2018). They study the transmission of the nominal interest rates to real lending rates in an economy with limited commitment, focusing on firm's financing choices. Whereas we focus on bank's portfolio choice. A recent paper by Amador and Bianchi (2024) models banks as borrowers that acquire liquidity through bond issuance. Their framework includes a stochastic-return capital asset as an outside option in the bank's asset structure. Their focus is on the bank's default decision, while ours is the relationship between pledgeability and intermediary contracts.

With nonmonotonic loan contract terms, our results bear on applications of the partial-equilibrium analysis. For example, Boot et al. (1991) predict that the interest-rate spread between risky and risk-free debt is monotonically declining in the value of collateral offered by the borrower. Indeed, the Federal Reserve Bank of Chicago's National Financial Conditions Index (NCFI) applies the monotonic relationship. In short, an increase in interest rate spreads, for example, is interpreted as tighter credit conditions.[6]

There is a large literature examining collateral in the optimal loan contract. Along the extensive margin, Hester (1972), Berger and Udell (1990,1995), and Klapper (1999) provide empirical support for the hypothesis that collateral is used by less creditworthy borrowers. Boot et al. (1991) explain collateral in loans in which the borrower has hidden action and hidden information. John et al. (2003) present evidence that there is a positive, extensive relationship between collateral and loan rates. They explain how lenders price agency risk into the loan rate. Along the intensive margin, Benmelech and Bergman (2009) argue that the empirical work suffers from selection bias inherent in collateral's extensive margin. In their paper, collateral redeployability as a proxy for the intensity of collateral pledged. They present evidence that there is a negative relationship between collateral values on both loan rates and loan size.[7]

[6]See Brave and Kelly (2017) for a complete description. The indicator is constructed using 105 variables, with 30 different interest rate spreads and another 28 variables that measure quantities or ratios. See also the theories presented by Boot et al. (1991)

[7]With both extensive and intensive margins, biased estimates are present if riskier firms are required to pledge more collateral. Because the quantity of collateral may be correlated with unobserved characteristics that affect loan rates, studies finding a positive relationship between loan rates and the presence of collateral are tainted by selection bias.

Outline

The rest of the paper is organized as follows. Section 2 provides empirical motivation. Section 3 presents the model environment. Section 4 solves the equilibrium in the competitive loan market. Section 5 considers Nash bargaining and competitive search in the loan market. Section 6 considers three extensions: lower entry costs, financial stability, and the effect of the Glass-Steagall Act. Section 7 concludes.

2 Empirical Motivation

In this section, we empirically examine how loan contract terms respond to the increase in pledgeability. Specifically, we focus on changes in loan rate and loan size trends before and after the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA).

Section 2.1 explains how BAPCPA improves the pledgeability of corporate bonds. Section 2.2 describes the data and details the construction of our sample. Section 2.3 presents aggregate loan patterns to visualize how loan rates and size have changed after the reform. Finally, Section 2.4 conducts event-study regressions to assess whether these changes are statistically significant.

2.1 Bankruptcy Abuse Prevention and Consumer Protection Act

Passed and implemented in 2005, BAPCPA is the most recent major reform of the U.S. Bankruptcy Code. While primarily targeting consumer bankruptcies, BAPCPA also enhances creditor protections in corporate bankruptcies. Specifically, BAPCPA improves the recovery of corporate bonds under Chapter 11, which is commonly used by firms seeking to reorganize. These enhancements are achieved through three main dimensions.[8]

First, BAPCPA imposed strict limits on the exclusivity period, during which only the

[8] Refer to Sprayregen et al. (2005) for a summary of all major corporate-side changes introduced by BAPCPA.

debtor may propose a reorganization plan. Previously, courts could extend this period for several years, leading to prolonged delays in bankruptcy proceedings. Under BAPCPA, the exclusivity period is capped at 120 days, encouraging faster resolutions and protecting creditors from delays that diminish the value of their claims.

Second, BAPCPA imposed restrictions on executive compensation and bonuses for firms in bankruptcy to address perceived abuses. Previously, companies could offer substantial bonuses to retain executives, often diverting resources away from creditors. The new law allows such payments only if specific conditions are met, such as the executive having a comparable job offer elsewhere, and caps their size relative to payments made to non-management employees.

Third, BAPCPA introduced measures to strengthen oversight and improve creditor rights. It extended the look-back period for avoiding fraudulent transfers from one to two years, increasing protections against improper asset transfers. Additionally, it expanded participation rights for noninstitutional creditors by granting them greater access to information and representation in committees. Lastly, BAPCPA requires the appointment of a trustee when there is suspicion of fraud or misconduct in the debtor's management, ensuring stronger oversight and accountability.

2.2 Data and Measures

Data Source. We use individual loan data from the Loan Pricing Corporation's (LPC) Dealscan database. Dealscan provides detailed information on the commercial loan market, including the dates and terms associated with loan issuances. Our focus is primarily on loan spreads and sizes, which are central to our model. Dealscan data are sourced from SEC filings, industry contacts, and lenders (Chava and Roberts, 2008). In our study, the unit of analysis is the loan (i.e., tranche), which is the most granular unit of observation in Dealscan.

Sample Selection. We apply three restrictions to our dataset. First, as BAPCPA was signed on April 20, 2005 and implemented on October 17, 2005, we limit our sample to

Table 1: Summary Statistics

	#obs	mean	sd	min	p25	p50	p75	max
Spread	18070	241.13	156.80	-95.00	150.00	225.00	300.00	1655.00
Loan size	21175	226.28	609.59	0.00	25.00	75.00	200.00	30000.00

Note: This table provides summary statistics for the Dealscan sample, where loan spreads are measured in basis points and loan sizes in millions of dollars. The sample is restricted to corporate loans with active dates from 2004 to 2007, originated within the United States, denominated in U.S. dollars, and issued as original loans rather than amendments.

loans with active dates between 2004 and 2007. This time frame allows us to analyze credit conditions during the relevant years leading up to, but excluding, the global financial crisis. This approach follows the one used by Gross et al. (2021). Next, we include only loans originating within the United States to be influenced by BAPCPA. We further restrict the sample to loans denominated in US dollars to exclude heterogeneity currency risks. Third, we include only original loan issuances, excluding loans that are amendments to existing agreements. Fourth, we focus on corporate loans, excluding loans of banks, government entities, media and communications, non-bank financial institutions, and utilities.[9]

Table 1 presents summary statistics for the set of loans selected. In our sample, approximately 20,000 loans were issued between 2004 and 2007. The data indicate substantial variation in both loan spreads and sizes. Loan spreads are calculated exclusively for loans that use LIBOR as the base reference rate, which applies to approximately 98.58% of the loans in the sample. Figure A.1 displays the LIBOR series, showing it remained stable around 5% during the sample period, with no notable shifts around the dates when BAPCPA was signed or implemented. Relative to LIBOR, the median loan spread in our sample is 225 basis points, with an interquartile range of 150 basis points; that is, two-thirds of the median.

The second row of Table 1 provides summary statistics for loan sizes. The median

[9]Corporate loans comprise approximately 74.28% of all loan types in our dataset.

loan size is $75 million. The mean is significantly higher at $226 million, indicating that the sample is skewed toward larger loans. The interquartile range for loan size is $175 million, more than twice the median, reflecting considerable variability in loan sizes across the sample.

2.3 Aggregate Patterns

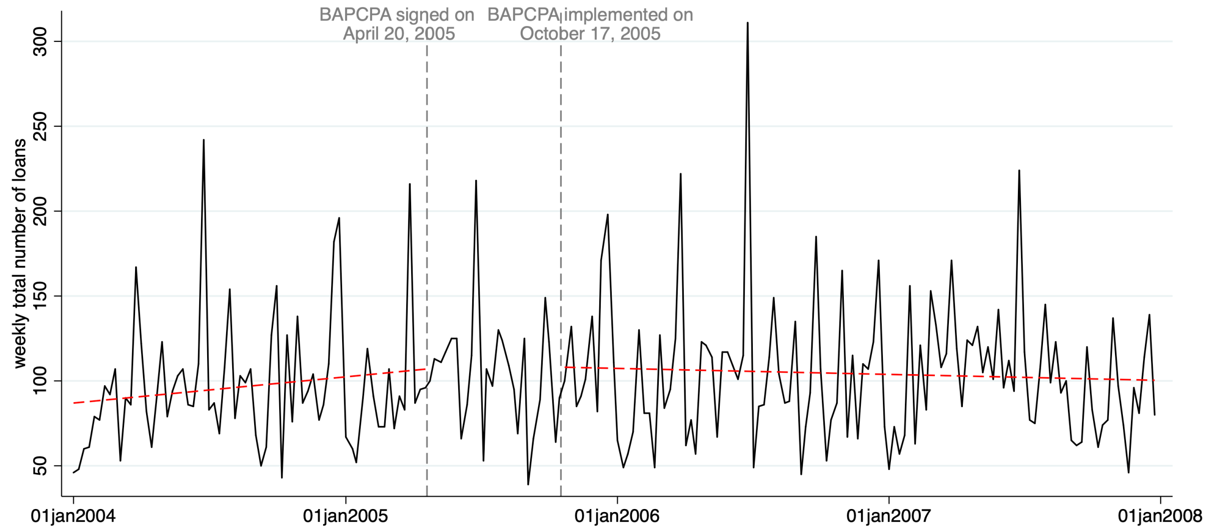
To examine changes in aggregate loan patterns following BAPCPA, we consolidate loan-level data into aggregate time series for visualization.

Figure 1 plots the total number of loans (Panel A) and the total loan amounts (Panel B) recorded in Dealscan from 2004 to 2007. To reduce noise, the daily panel is aggregated into weekly frequencies based on active loan dates. The two vertical dashed gray lines mark the passage of BAPCPA on April 20, 2005, and its implementation on October 17, 2005. In each panel, we plot two red fitted lines: one for the period before BAPCPA was passed and another for the period after its implementation. These fitted lines are linear predictions of the variable of interest over time.

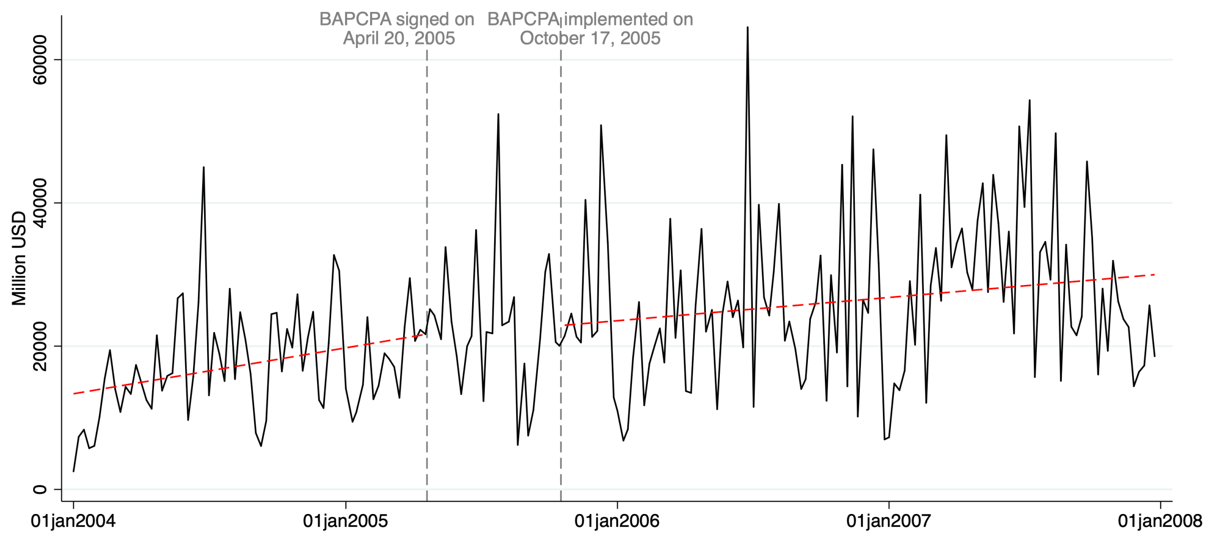
Panel A shows a noticeable shift in the number of loans issued before and after BAPCPA, while Panel B depicts corresponding changes in loan volumes. The evidence suggests that the growth rates of both loan issuances and volumes declined following the enactment of BAPCPA. Although Dealscan may not encompass the entire loan market, as long as BAPCPA does not significantly change Dealscan's coverage, its aggregate patterns remain informative.

Figure 2 presents the time series of weighted average loan spreads (Panel A) and weighted average loan sizes (Panel B), with weights constructed using relative loan sizes.[10] We also aggregate the daily averages into a weekly frequency according to the active dates of the loans here. Panel A shows a clear downward trend in (weighted) loan spreads before the passage of BAPCPA. However, on BAPCPA's implementation, we do not observe a decrease in loan spreads. Instead, the data indicate that the decline

[10]Figure A.2 in the appendix plots the unweighted averages, which show similar patterns.



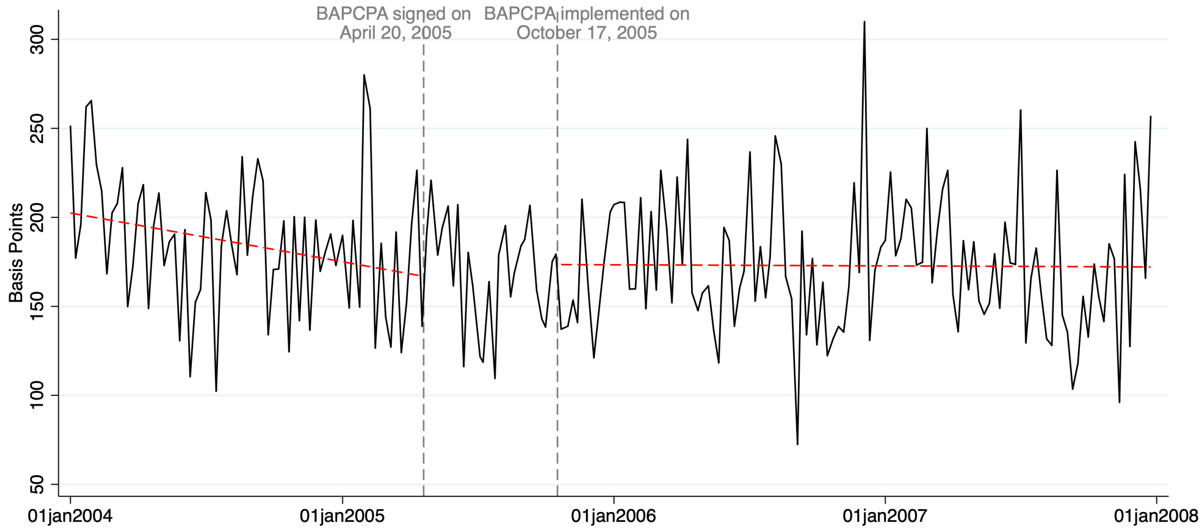
Panel A. Total Number of Loans



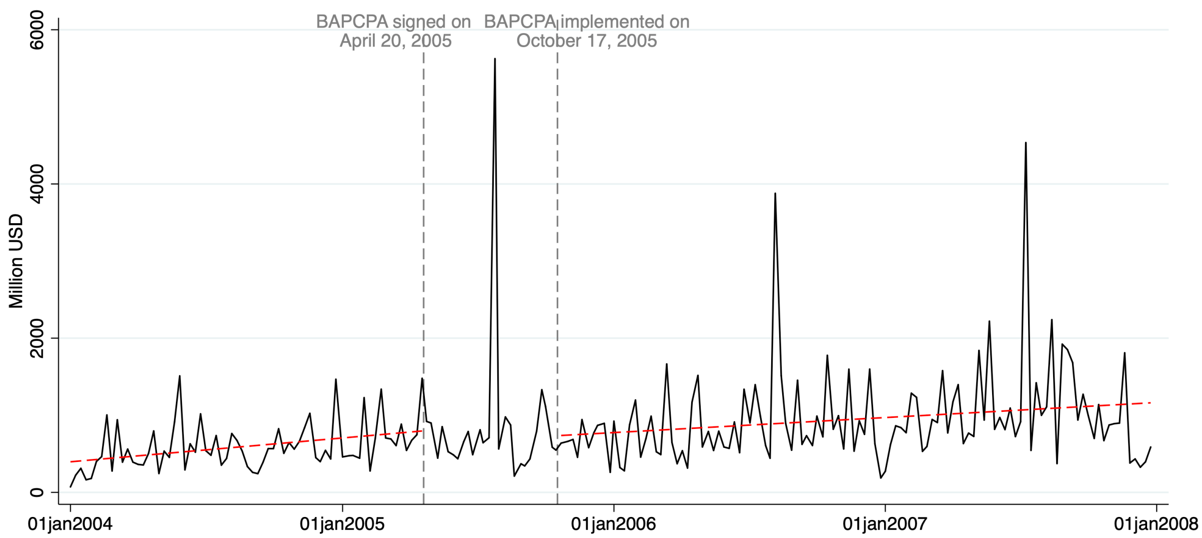
Panel B. Total Amount of Loans

Figure 1: Weekly Time Series of Total Loans

Note: This figure shows the total number of loans (Panel A) and the total loan amounts (Panel B) in Dealscan from 2004 to 2007. We aggregate the loan-level data into a weekly frequency based on the active dates of the loans. The two vertical dashed gray lines mark the passage of BAPCPA on April 20, 2005, and its implementation on October 17, 2005. In each panel, we plot two red fitted lines: one for the period before BAPCPA was passed and another for the period after its implementation. These fitted lines are linear predictions of the variable of interest over time.



Panel A. Weighted Average Spreads of Loans



Panel B. Weighted Average Sizes of Loans

Figure 2: Weekly Weighted Average Spreads and Sizes of Loans

Note: This figure shows the weighted average spreads of loans (Panel A) and the weighted average loan sizes (Panel B) in Dealscan from 2004 to 2007, with weights based on relative loan sizes. We aggregate the daily averages into a weekly frequency based on the active dates of the loans. The two vertical dashed gray lines mark the passage of BAPCPA on April 20, 2005, and its implementation on October 17, 2005. In each panel, we plot two red fitted lines: one for the period before BAPCPA was passed and another for the period after its implementation. These fitted lines are linear predictions of the variable of interest over time.

in loan spreads even came to a complete halt following the law's implementation.

Panel B of Figure 2 plots the weekly weighted average loan sizes. While the trends in weighted loan sizes remain positive before and after BAPCPA's implementation, the post-BAPCPA trend is consistently lower than the trajectory predicted by the pre-BAPCPA trend. This suggests a reduction in loan sizes following BAPCPA.[11]

These two patterns point to a significant shift in loan contracts, both in price and quantity, after the implementation of BAPCPA. Importantly, these observed changes are not predicted by conventional logic regarding the increase in pledgeability.

2.4 Event-Study Regressions

We then proceed to a more formal statistical analysis. Were the differences in trends statistically significant when comparing loan spreads and loan sizes before and after BAPCPA? To assess the statistical significance of the changes that occur after the passage of BAPCPA, we perform event-study regressions. Our identifying assumption is as follows: in the absence of the bankruptcy reform, both the level and the trends observed prior to the week of April 20, 2005—when BAPCPA passed—would have been constant over time.

Consider the following regression specification:

$$y_{it} = \beta_0 + \beta_1(t - \bar{t}) + \beta_2 \mathbb{1}\{t \geq \bar{t}\} + \beta_3(t - \bar{t}) \times \mathbb{1}\{t \geq \bar{t}\} + \epsilon_{it}, \tag{1}$$

where the dependent variable y_{it} represents the characteristic of loan i issued at time t, which can be either the loan's spread or size. Here, $\bar{t}$ denotes April 20, 2005, the reference date that marks the passage of BAPCPA.[12] The term $t - \bar{t}$ captures the distance in time from this reference point, equal to zero on April 20, 2005; negative values before

[11] Panel B of Figure A.2 in the appendix shows the unweighted average loan sizes, where the change is less pronounced, indicating that the reduction in loan sizes primarily exists among larger loans.

[12] We follow Gross et al. (2021) in using the passage of BAPCPA as the cutoff. As they point out, under the bankruptcy code, debts incurred in the months leading up to a filing are not eligible for discharge. As a result, loans issued between passage and implementation were unlikely to be discharged under the old, more lenient rules.

Table 2: Event Study for Loan Spreads and Sizes

	Spread		log(Size)	
	(1)	(2)	(3)	(4)
$\hat{\beta}_0 : 1$	244.60***	168.64***	4.42***	6.00***
	(4.32)	(7.47)	(0.04)	(0.07)
$\hat{\beta}_1 : t - \bar{t}$	-13.40**	-25.54**	0.12**	0.58***
	(5.86)	(10.80)	(0.05)	(0.10)
$\hat{\beta}_2 : 1\{t \geq \bar{t}\}$	-7.50	0.07	-0.07	-0.23**
	(5.14)	(9.02)	(0.05)	(0.09)
$\hat{\beta}_3 : (t - \bar{t}) \times 1\{t \geq \bar{t}\}$	12.61**	28.39**	-0.03	-0.45***
	(6.13)	(11.28)	(0.06)	(0.11)
Observations	18070	18070	18070	18070
R^2	0.003	0.003	0.003	0.024
Wald test p-value for $\beta_1 + \beta_3 = 0$	0.66	0.38	0.00	0.00
Weighted by loan size	×	✓	×	✓

Note: This table presents the event study results for the following regression specification: $y_{it} = \beta_0 + \beta_1(t - \bar{t}) + \beta_2 \mathbb{1}\{t \geq \bar{t}\} + \beta_3(t - \bar{t}) \times \mathbb{1}\{t \geq \bar{t}\} + \epsilon_{it}$, where the dependent variable y_{it} represents the characteristic of loan i issued at time t, which can be either the loan's spreads (Columns 1 and 2) or size (Columns 3 and 4). The unit for loan spreads is basis points. The explanatory variable $\bar{t}$ denotes April 20, 2005, the reference date marking the passage of BAPCPA. The term $t - \bar{t}$ captures the distance in time from this reference point, and we annualize it by dividing by 365. The indicator function $\mathbb{1}\{t \geq \bar{t}\}$ equals one if the loan is issued post-BAPCPA. The residual term is denoted as ϵ_{it}. The second-to-last row displays the p-values from the Wald test for $\beta_1 + \beta_3 = 0$, which assesses the growth rate of loan spreads after the passage of BAPCPA. Regressions in Columns 1 and 3 are unweighted, while those in Columns 2 and 4 are weighted by loan sizes. Standard errors, shown in parentheses, are robust. Statistical significance is indicated by stars: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

this date; and positive values afterward. To facilitate the interpretation of coefficients, we annualize $t - \bar{t}$ by dividing it by 365. The indicator function $\mathbb{1}\{t \geq \bar{t}\}$ equals one if the loan is issued post-BAPCPA. The residual term is denoted as ϵ_{it}. In this model, β_2 measures the immediate change in the loan characteristic on the day BAPCPA was passed, while β_3 indicates the change in trend following the passage of the reform.

Table 2 presents the results of the event-study regressions, which are consistent with the aggregate patterns observed in Figure 2.[13] In the first two columns, the dependent variable is the loan spread measured in basis points. Column 1 reports the results from an unweighted regression. The estimated coefficient of $t - \bar{t}$ is significantly negative, indicating that loan spreads decreased, on average, by 13.40 basis points per year prior to the passage of BAPCPA. In contrast, the estimated coefficient of $\mathbb{1}\{t \geq \bar{t}\}$ is statistically insignificant, suggesting that the law did not reduce loan spreads upon its passage. Importantly, this lack of impact cannot be attributed to lagged effects, as the coefficient of the interaction term $(t - \bar{t}) \times \mathbb{1}\{t \geq \bar{t}\}$ is significantly positive, with a sizable magnitude of 12.61. This indicates that loan spreads, in fact, had an increasing trend over time due to BAPCPA.

Column 2 presents results from regressions weighted by loan sizes, revealing a larger decline in loan spreads before BAPCPA and a more pronounced increase afterward. Still, there is no evidence of a significant decline in loan spreads at the time of BAPCPA implementation, as the coefficient β_2 is statistically indistinguishable from zero. Regarding the post-BAPCPA trend, the second-to-last row shows the p-values from the Wald test for $\beta_1 + \beta_3 = 0$, which assesses the growth rate of loan spreads after the passage of BAPCPA. The large p-values indicate that this growth rate is statistically insignificant from zero, suggesting that the previously declining trend in loan spreads effectively ceased following the passage of BAPCPA.

Columns 3 and 4 replace the dependent variable with the logged loan sizes. Both the unweighted regression (Column 3) and the weighted regression (Column 4) show significant and positive coefficients for $t - \bar{t}$, indicating an increase in the average loan sizes prior to the reform. Specifically, the unweighted pre-BAPCPA growth rate is 12% per year, and the weighted pre-BAPCPA growth rate is even higher at 58% per year. These results highlight the expansion of the loan market prior to BAPCPA.

However, the passage of BAPCPA disrupted this upward trend, particularly for larger loans. In the weighted regression (Column 4), the estimated coefficient β_2 is

[13]See also A.2 in the appendix

significantly negative, suggesting that the loan sizes immediately decreased by 24% upon the passage of BAPCPA. Furthermore, the significantly negative coefficient of -0.45 for the interaction term $(t - \bar{t}) \times \mathbb{1}\{t \geq \bar{t}\}$ indicates a 45% slowdown in the growth rate of loan sizes after the reform. Since the unweighted regression does not show significant changes (Column 3), these findings imply that the discontinuity is primarily concentrated among the larger loans.

The results of the event study reject the null hypothesis of no break in the loan market following the passage of BAPCPA. More importantly, these findings are not predicted by conventional logic regarding the effects of increased pledgeability. Specifically, the observed loan spreads did not decrease and the loan sizes did not increase in response to BAPCPA. Motivated by these observations, in the following sections, we develop a general equilibrium model to explore how increased pledgeability affects loan contracts, offering a framework to can account for this puzzle.

3 The Model

The model environment is based on Diamond-Dybvig with the introduction of an additional type of agent: borrowers. There are three time periods indexed by $t = 0, 1, 2$. There are two types of agent in this economy: depositors and borrowers. The measure of depositors is normalized to 1 while the measure of borrowers is n. Each depositor is endowed with one unit of capital in $t = 0$ and nothing in periods $t = 1, 2$. Depositors have access to a long-term investment technology that turns 1 unit of capital at $t = 0$ into $\underline{R} > 1$ units of consumption good at $t = 2$ or 1 unit of consumption good at $t = 1$. There is also a storage technology that can transform capital into consumption good at a one-for-one rate in either period.

Depositors are identical in period 0. At date $t = 1$, each depositor receives an idiosyncratic preference shock. With probability λ, the depositor is impatient, meaning he derives utility exclusively from consuming in $t = 1$. With probability $1 - \lambda$, the depositor is patient, meaning he values consumption in $t = 2$.

Let $u(x_t)$ denote the utility function of the depositor, where x_t is the consumption in period t. The utility function is strictly increasing, strictly concave, and normalized to $u(0) = 0$. The coefficient of relative risk aversion (CRRA), $-xu''(x)/u'(x)$, is greater than 1 for $x \geq 1$. Whether a depositor is patient or impatient is his private information. By the law of large numbers, λ is also the fraction of depositors in the population who are impatient.

Borrowers have access to a technology that turns 1 unit of capital at $t = 0$ into $\bar{R} > \underline{R}$ units of consumption good at $t = 2$ or one unit of consumption good at $t = 1$. They are not endowed with capital. However, they can produce capital by incurring a utility cost at $t = 0$. Let $c(k)$ be the cost function of producing k units of capital at $t = 0$, where $c', c'' > 0$ and $c'(0) = 0$. Borrowers consume at $t = 2$ with utility function $v(x)$, where $v' > 0 > v''$.

Following Diamond-Dybvig, depositors have the incentive to form a coalition that acts as a bank by providing themselves with a deposit contract to insure against the consumption shock. The borrowers have access to better technology. So, if there is no further friction, banks should lend their capital to borrowers in $t = 0$ and get a higher return. However, borrowers lack commitment. When a borrower receives a loan, he can liquidate the investment at $t = 1$, transforming the capital into the consumption good at a one-for-one rate, repaying only the pledged fraction of the collateral posted. Here, posted collateral consists of the sum of loans and the borrower's capital. For simplicity, we assume that if the borrower reneges on the loan, he absconds with the unpledgeable collateral. Let χ be the fraction of the collateral collected by the bank, so $1 - \chi$ is the fraction consumed by the borrower if he reneges on the loan.[14]

In this environment, banks and borrowers can work together to economize on investment and take advantage of the higher-return technology. How gains from trade are determined and divided will depend on the market structure. To develop intuition, we first study a competitive loan market in which banks and borrowers take the loan

[14]This is modeled as cash diversion in Biais et al. (2007) and DeMarzo and Fishman (2007).

rate as given and choose the size of the loan. We then consider bilateral trade, where the terms of trade are determined according to Nash bargaining. Lastly, we consider competitive search, so the trade surplus is divided according to the market tightness.

To set up a benchmark for comparison, we first calculate the payoffs of banks and borrowers without the loan market. As a coalition of depositors, the bank seeks to maximize the expected welfare of its depositors. Formally, the deposit contract solves

$$\hat{W}_D = \max_{x_1, x_2} \left[\lambda u(x_1) + (1-\lambda) u(x_2)\right] \tag{2}$$

$$\text{st } (1-\lambda x_1) \underline{R} = (1-\lambda) x_2 \tag{3}$$

$$x_2 \geq x_1 \tag{4}$$

where (3) is the resource constraint and (4) is the incentive constraint for late depositors to wait until $t = 2$ to withdraw. The bank liquidates λx_1 from its investment in $t = 1$ to pay impatient depositors and leave the rest until $t = 2$ with return $\underline{R}$ to pay patient ones. As standard, the solution to (2), denoted by $(\hat{x}_1, \hat{x}_2)$ satisfies the first-order condition $u'(x_1) = \underline{R} u'(x_2)$ and (3). At $(\hat{x}_1, \hat{x}_2)$, (4) does not bind.

Without the loan market, borrowers solve:

$$\hat{W}_B = \max_k \left[-c(k) + v\left(\bar{R}k\right)\right]$$

The first-order condition is $c'(k) = \bar{R} v'\left(\bar{R}k\right)$. Let the solution for capital be denoted by $\hat{k}$. Borrowers consume $\hat{x}_B = \bar{R}\hat{k}$.

4 Competitive Loan Market – A Baseline

In a competitive loan market, both banks and borrowers take the market loan rate as given and choose the size of the loan to maximize their expected utility. The bank's problem is

$$\max_{x_1,x_2,a} \left[\lambda u(x_1) + (1-\lambda) u(x_2)\right]$$
$$\text{st } (1-\lambda) x_2 = (1-\lambda x_1 - a) r + a\underline{R} \tag{5}$$

and (4), where r is the market loan rate. The bank keeps λx_1 in storage and invests a in its own long-term technology.[15] Thus, $1-\lambda x_1 - a$ is the amount of the loan extended to the borrowers. Equation (5) is the resource constraint for the bank. The bank expects the loan to be paid at $t = 2$ with the interest rate r. From its own production, the bank pays impatient depositors λx_1 at $t = 1$ and the safe-haven investment a, matures with return $\underline{R}$. With these resources, the bank pays the patient depositors at $t = 2$.

The first-order condition with respect to x_1 is

$$u'(x_1) - ru'(x_2) = 0 \tag{6}$$

By (5), $a = 0$ if $r > \underline{R}$, $0 < a < 1 - \lambda x_1$ if $r = \underline{R}$, and $a = 1 - \lambda x_1$ if $r < \underline{R}$.

The borrower's problem is

$$\max_{k,\ell,x_B} \left[-c(k) + v(x_B)\right]$$
$$\text{st } x_B = \left(\bar{R} - r\right)\ell + \bar{R}k \tag{7}$$
$$x_B \geq (1-\chi)(\ell + k) \tag{8}$$

where ℓ is the loan amount. Equation (7) is the borrower's resource constraint. His consumption is financed by two sources: he borrows ℓ from the bank, invests it, gets return $\bar{R}$ and pays r; in addition, his own capital yields $\bar{R}$. The borrower's repayment constraint is described by (8). The borrower can abscond with $1-\chi$ fraction of the investment, and the bank recovers χ fraction of it. The contract must make sure that the borrower's equilibrium payoff cannot be less than the deviation payoff.

[15]It does not matter whether the bank keeps λx_1 in storage or in its own long-term technology because these two options yield the same retrun at $t = 1$.

The first-order conditions are

$$-\left(r-\bar{R}+1-\chi\right)c'(k)+r(1-\chi)v'(x_B) = 0 \tag{9}$$

$$\eta-\frac{\left(\bar{R}-r\right)v'(x_B)}{r-\bar{R}+1-\chi} = 0 \tag{10}$$

where η is the Lagrangian multiplier associated with (8); in other words, the shadow value of the borrower's repayment constraint. Note that for $r<\bar{R}-1+\chi$, there will be infinite demand for ℓ. So r cannot be lower than $\bar{R}-1+\chi$. There are two cases, depending on whether (8) binds. First, with $r=\bar{R}$, then $k=\hat{k}$ and $\ell<\left(\bar{R}-1+\chi\right)\hat{k}/(1-\chi)$. Second, with $\bar{R}-1+\chi\leq r<\bar{R}$, the demand for loans is solved from (9) with binding (8). That is,

$$\left(r-\bar{R}+1-\chi\right)c'\left(\frac{r-\bar{R}+1-\chi}{\bar{R}-1+\chi}\ell\right)=r(1-\chi)v'\left(\frac{r(1-\chi)}{\bar{R}-1+\chi}\ell\right)$$

Finally, the loan-market clearing condition is

$$1-\lambda x_1-a=n\ell \tag{11}$$

which solves for equilibrium r.

The equilibrium contract is in one of the three distinct regions, depending on whether (8) binds and whether $a>0$ or $a=0$. We refer to the High-pledgeability Region as the range of χ values where (8) does not bind, the Medium-pledgeability Region as the range where (8) binds and $a=0$, and the Low-credibility Region as the range where (8) binds and $a>0$. We discuss each case.

1. High-pledgeability Region: With $\eta=0$, we solve (5)-(11) to get $a=0$, $r=\bar{R}$, $k=\hat{k}$, $x_1=x_1^*$ and $x_2=x_2^*$, where $\left(x_1^*,x_2^*\right)$ satisfies $u'(x_1)=\bar{R}u'(x_2)$ and $(1-\lambda x_1)\bar{R}=(1-\lambda)x_2$. The aggregate loan size is $1-\lambda x_1^*$. Each borrower gets $\ell^*=\left(1-\lambda x_1^*\right)/n$. The economy is in this region if and only if $\chi\geq\bar{\chi}\equiv 1-\hat{k}\bar{R}/\left(\ell^*+\hat{k}\right)$.

   In the High-pledgeability Region, the bank takes full advantage of the borrower's technology. The marginal rate of substitution between x_1 and x_2 equals the marginal rate of transformation of the borrower's technology. The borrower's capital production and consumption are the same as in autarky.

2. Medium-pledgeability Region: With $\eta > 0$ and $a = 0$, the Medium Region is associated with values of $\underline{\chi} \leq \chi < \bar{\chi}$, where $\underline{\chi}$ is defined in the proof of Proposition 1 in the Appendix. In this region, $\underline{R} \leq r < \bar{R}$.

   In the Medium Region, r and χ are positively related. When χ increases, borrowers are able to borrow more, which intensifies competition for loans. This competition drives up the interest rate. Following Diamond-Dybvig, depositor's coefficient of relative risk aversion (CRRA) is bigger than 1. A high CRRA implies a low elasticity of intertemporal substitution. Therefore, with a higher r, depositors consume more in both periods and banks lend less, meaning the loan supply curve slopes downward. Consequently, the loan size decreases with χ. However, the loan size is higher than in the High-pledgeability Region to partially offset the loss in the return per unit of investment.

   It is not clear if the borrowers produce more capital compared with the High-pledgeability Region. Two countervailing forces are at work. On the one hand, as assets are less pledgeable, the loan contract requires more collateral to induce the borrower to repay the loan. On the other hand, since r is lower in the Medium Region, the unit repayment cost is less, which helps relax the repayment constraint. It turns out that the borrower's elasticity of intertemporal substitution, $-v'/xv''$, determines the sign of the overall effect. With high elasticity, the borrowers reduce the production of collateral as the opportunity cost becomes higher when r increases, and vice versa for low elasticity.

   Similarly, x_B is not necessarily monotone in χ. But we show it is strictly decreasing at $\bar{\chi}$ so is the borrower's overall utility. To understand this, assume first that the market rate r stays constant as χ increases from $\bar{\chi}$. Borrowers choose ℓ and k

Table 3: Comparative Statics, Competitive Market

	$dx_1/d\chi$	$dx_2/d\chi$	$dx_B/d\chi$	$dk/d\chi$	$dr/d\chi$	$d\ell/d\chi$	$da/d\chi$
High	0	0	0	0	0	0	N.A.
Medium	+	+	±	±	+	−	N.A.
Low	0	0	+	±	0	+	−

along the the envelope frontier of $-c + v$ as (8) just binds. No first-order effect is at work. However, r is not constant. It rises as borrowers compete for loans when the repayment constraint is relaxed. The change in r has a first-order effect: it means that the borrowing cost is higher.

3. Low-pledgeability Region: With $\eta > 0$ and $a > 0$, we have that $r = \underline{R}$. It follows immediately that $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$ as in autarky. The borrowers take full advantage of the loan market as they pay the bank's reservation rate. This region requires $\chi < \underline{\chi}$. In the Low-pledgeability Region, ℓ and x_B are both strictly increasing in χ and a is strictly decreasing. Again, we find that whether k increases or decreases depends on $-xv''/v'$.

The result is summarized in the following proposition.

Proposition 1 *Consider a perfectly competitive loan market. There exist $\underline{\chi}$ and $\bar{\chi}$, with $\underline{\chi} < \bar{\chi}$ such that (1) if $\chi \geq \bar{\chi}$, the equilibrium is in the High-pledgeability Region; (2) if $\underline{\chi} \leq \chi < \bar{\chi}$, the equilibrium is in the Medium-pledgeability Region; (3) if $\chi < \underline{\chi}$, the equilibrium is in the Low-pledgeability Region.*

For each of the three plegeability regions, the comparative statics are summarized in Table 3. Here, $dk/d\chi \doteq v'/v''x + 1$ in the Medium Region, where $\doteq$ means the same sign. In the Low Region, $dk/d\chi \doteq -(v'/v''x + 1)$ and $dx_B/d\chi|_{\bar{\chi}} < 0$.

Consider a case in which χ increases from 0. The borrowing conditions change in several dimensions: collateral, loan rate, and loan size. Note that with $\chi = 0$, banks can still lend some positive amount to borrowers, because the borrower would have to give up the return on his own capital if he absconds. As χ increases, the borrowers become less constrained and borrow more. Banks engage less in direct investment, the loan rate stays at $\underline{R}$, which is the same as the return on direct investment, and k may rise or fall. As χ crosses $\underline{\chi}$, banks no longer invest in the low-return technology. The competition for loans eventually raises the loan rate. The loan size decreases. The relationship between k and χ is reversed. As χ increases further and crosses $\bar{\chi}$, the repayment constraint no longer binds. The loan rate stays at $\bar{R}$, the loan size stays constant, and k stays at $\hat{k}$.

To illustrate the equilibrium outcomes for different values of collateral pledgeability, we turn to numerical analysis. The following functions and parameters are used for the experiments. The results are plotted in Figure 3.

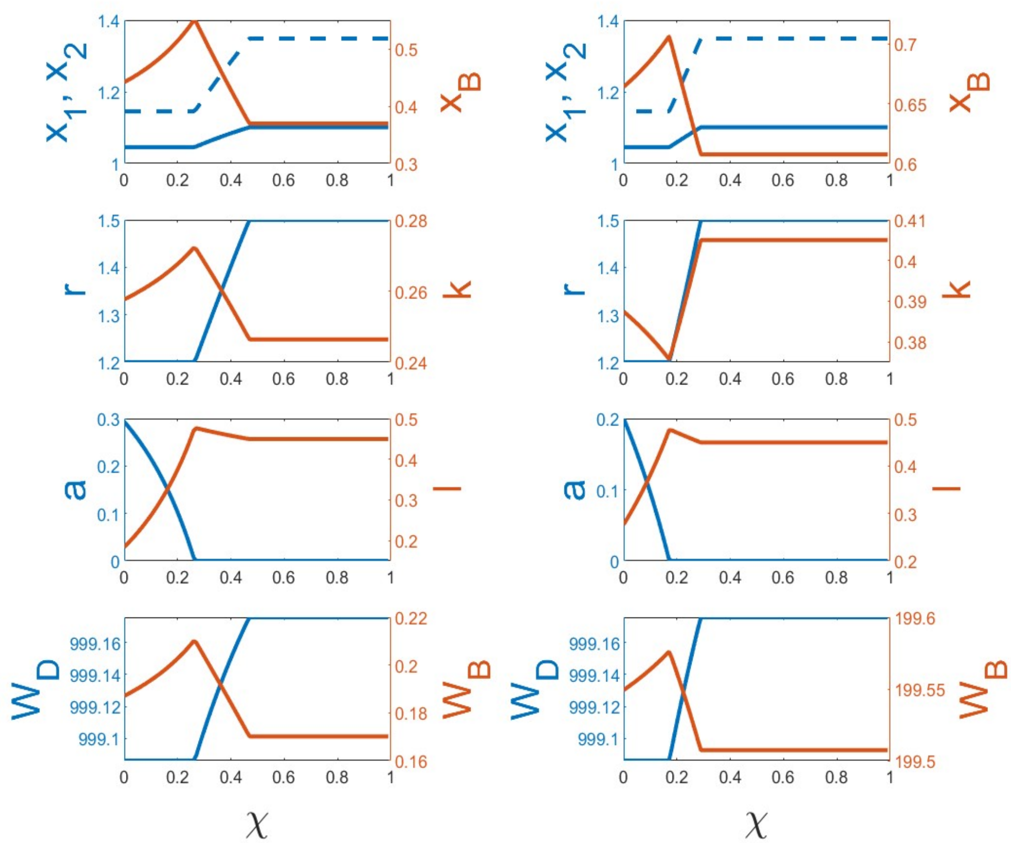


Figure 3: Competitive Equilibrium, $\gamma > 1$. Left: $\delta = 0.5$; Right: $\delta = 2$.

Let

$$u(x) = \frac{(x+b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma}, c(k) = Bk^{\alpha}, v(x) = A\frac{(x+b)^{1-\delta} - b^{1-\delta}}{1-\delta}$$

where $b = 0.001$, $\gamma = 2$, $B = 1$, $\alpha = 2$ and $A = 0.2$. Other parameters are $\bar{R} = 1.5$, $\underline{R} = 1.2$, $\lambda = 0.5$, and $n = 1$. The left column uses $\delta = 0.5$, and the right column uses $\delta = 2$.

With $\delta = 0.5$, $\bar{\chi} = 0.47$ and $\underline{\chi} = 0.27$. As $-xv''/v' = \delta < 1$, k first increases in the Low Region and then decreases in the Medium Region. With $\delta = 2$, $\bar{\chi} = 0.29$ and $\underline{\chi} = 0.18$. Capital first decreases in the Low Region and then increases in the Medium Region. In both examples, loan size, borrower's consumption, and borrower's welfare first increase, then decrease, and eventually stay constant. In particular, the right panel shows a case where collateral intensity (i.e., k/ℓ ratio) rises as assets become more pledgeable in the Medium Region. The loan rate is constant in the Low and High Regions and strictly increasing in the Medium Region. On the deposit contract side, both types of depositor consume more as χ increases, but x_2 increases faster than x_1. Although risk sharing is weakened, the overall welfare of depositors improves.

Risk aversion can explain why loan size is not monotonically related to pledgeability. In Diamond-Dybvig, the deposit contract provides partial risk sharing to depositors. A more risk averse depositor seeks to smooth consumption across the two possible states. Indeed, the depositor must be risk averse enough so that $x_1 > 1$. Under such a deposit contract, banks are subject to panic runs. If depositors are less risk averse, that is, $CRRA \leq 1$, they will prefer a more volatile consumption profile that pays $x_1 \leq 1$ and $x_2 \geq \bar{R}$. Risk aversion is negatively related to intertemporal elasticity of substitution. With $CRRA \leq 1$, the elasticity of substitution is so high that the loan supply increases in the loan rate. Figure 4 plots examples using the parameters in Figure 3 except that $\gamma = 0.5$. As χ increases, depositors consume less x_1 and more x_2 in the Medium Region, and the loan size monotonically increases across Regions.

Notice that the pattern of r and ℓ in the Medium Region with depositor's $CRRA > 1$ is consistent with our empirical evidence on BAPCPA. Specifically, the evidence

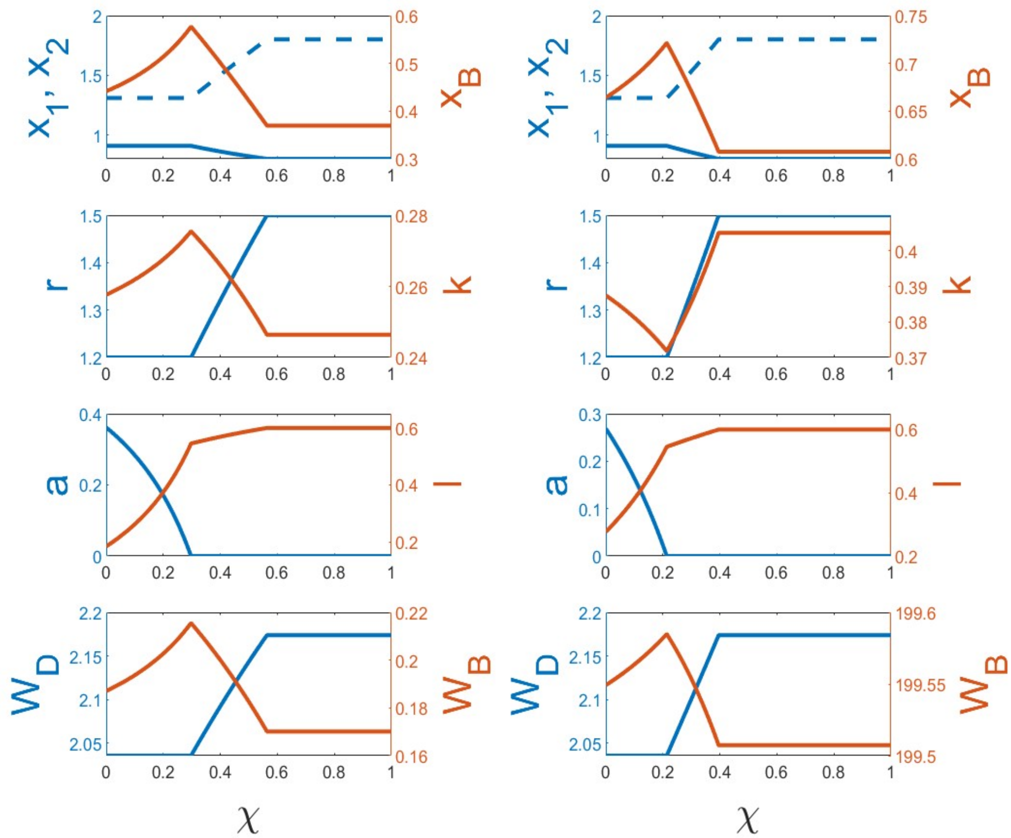


Figure 4: Competitive Equilibrium, $\gamma < 1$. Left: $\delta = 0.5$; Right: $\delta = 2$.

suggests that the loan size decreases following the passing of the law. Although the loan rate shows no significant immediate change, there is evidence supporting the notion that loan rates are above the pre-law trend and that the gap is widening over time. Studying dynamics of the contract terms is outside our model. However, our model can capture the structural break and correctly predicts the direction of these changes.

The results of our baseline model show that the relationship between the contract terms and the pledgeability of the asset is more complicated than conventional wisdom would have us believe. In general, the contract terms are nonmonotonic in pledgeability, and the patterns are sensitive to the elasticity of intertemporal substitution of agents on both sides of the market.

5 Market Structure

In this section, we test the robustness of the nonmonotonicity results by examining the market response to changes in pledgeability under different loan market structures.

5.1 Nash Bargaining

First, we consider bilateral trade. The idea is that this market structure is a way to capture features of an OTC market. Banks are coalitions of depositors. Banks and borrowers meet bilaterally to decide the terms of the contract. If one or both parties do not agree on the terms of trade, the bank and the borrower walk away and receive the payoff $\hat{W}_D$ and $\hat{W}_B$, respectively. Otherwise, they jointly determine consumption, capital, loan size, and rate according to the generalized Nash bargaining solution. Let θ be the bargaining power of the bank. The generalized Nash problem is

$$\max_{x_1,x_2,x_B,r,k,a} \left[\lambda u(x_1) + (1-\lambda) u(x_2) - \hat{W}_D\right]^{\theta} \left[-c(k) + v(x_B) - \hat{W}_B\right]^{1-\theta} \tag{12}$$

$$\text{st (4), (5)}$$

$$x_B = (1-\lambda x_1 - a)\left(\bar{R} - r\right) + k\bar{R} \tag{13}$$

$$x_B \geq (1-\chi)(1-\lambda x_1 - a + k) \tag{14}$$

The first-order conditions with respect to (x_1, r, k, a) simplify to

$$u'(x_1)\left[c'(k) - \chi v'(x_B)\right] - \left(\bar{R} - \chi\right) u'(x_2) c'(k) = 0 \tag{15}$$

$$a\left[-u'(x_1) + \underline{R} u'(x_2)\right] = 0 \tag{16}$$

$$\theta u'(x_1) S_B - (1-\theta) c'(k) S_D = 0 \tag{17}$$

$$\chi\eta - \frac{(1-\theta) S_D c'(k)}{u'(x_1)}\left[\bar{R} u'(x_2) - u'(x_1)\right] = 0 \tag{18}$$

where the bank's trade surplus is $S_D \equiv \lambda u(x_1) + (1-\lambda) u(x_2) - \hat{W}_D$.[16] The borrower's trade surplus is $S_B \equiv -c(k) + v(x_B) - \hat{W}_B$. Finally, η is the Lagrangian multiplier associated with (14). As we found in the competitive market, there are three regions divided according to χ. Abusing notation, we use $\underline{\chi}$ and $\bar{\chi}$ to denote the cutoffs between the regions.

1. High-pledgeability Region: With $\eta = 0$ and $a = 0$, the High-pledgeability Region is characterized by $u'(x_1)/u'(x_2) = c'(k)/v'(x_B) = \bar{R}$. This condition implies that the bank and the borrower exclusively utilize the borrower's higher-return technology and that the allocation is efficient. However, as $\theta \in (0,1)$, the bank does not get all surplus. Thus, $r < \bar{R}$. Denote the solution by $\left(\tilde{x}_1, \tilde{k}, \tilde{r}\right)$. The contract is in the High Region if and only if $\chi \geq \bar{\chi} \equiv 1 - \bar{R} + \tilde{r}(1-\lambda\tilde{x}_1)/\left(1-\lambda\tilde{x}_1+\tilde{k}\right)$.

2. Medium-pledgeability Region: With $\eta > 0$ and $a = 0$, the contract is in the Medium Region if and only if $\underline{\chi} \leq \chi < \bar{\chi}$. The proof of the existence and uniqueness of $\underline{\chi}$ is in the appendix. Here, $\underline{R} \leq u'(x_1)/u'(x_2) < \bar{R}$, which implies the bank also undertakes the cost of twisting the investment in k by reducing the loan size and/or the rate, resulting in the inefficiency in the marginal rate of intertemporal substitution. On the borrower's side, $c'(k) > \bar{R}v'(x_B)$, which implies that the marginal investment in k not only adds more consumption but also relaxes the borrowing constraint.

3. Low-pledgeability Region: With $\eta > 0$ and $a > 0$, the loan contract is in the Low-pledgeability Region if and only if $\chi < \underline{\chi}$. Again, $c'(k) > \bar{R}v'(x_B)$ as adding capital relaxes (14). On the bank's side, $u'(x_1) = \underline{R}u'(x_2)$ as the marginal investment of the bank yields $\underline{R}$.

[16]We rule out some possible corner solutions. First, $\ell = 1 - \lambda x_1 - a > 0$, which means that the bank always invests in the borrower. By setting $\ell = 0$, the two parties are in autarky and (4) and (14) do not bind. With r slightly below $\bar{R}$ and offering loans slightly greater than zero, all constraints remain satisfied, and both banks and borrowers have slightly more resources. Hence, ℓ is strictly positive. Next, note that (4) does not bind because regardless of the bank's investment portfolio, the return in $t = 2$ is bounded below by $\underline{R} > 1$. So, it is costly for patient consumers to liquidate early. Finally, $k > 0$ as $v'(x_B) > c'(0) = 0$.

The contract terms once again exhibit nonmonotonic patterns, but for a different reason. In bilateral trade, there is no demand or supply curve, so these patterns are not driven by the elasticity of intertemporal substitution. Instead, both parties fully internalize all terms of the contract, including r, to adapt to changes in χ. The marginal value of each term is different when the repayment constraint binds. Hence, the terms may not change in the same magnitude or even in the same direction with the change in χ.

The Nash bargaining solution has some interesting properties. As Kalai (1977) showed, one's surplus does not necessarily increase with the expansion of the bargaining set. Because of this property, it is hard to derive unambiguous comparative statics for all variables.

There is one clear comparative static. The quantity of direct investment is inversely related to pledgeability in the Low Region. Because the borrower's project return dominates the risk-free rate, the safe asset is the last tool that the bank uses to cope with the deteriorating pledgeability conditions. The intuition is the same as in the baseline model: adjusting variables such as k, r, and ℓ moves the economy along the envelope frontier and does not have a first-order impact. However, increasing a from 0 reduces the resources allocated to productive investment and shrinks the consumption set. To summarize,

Proposition 2 *Under the generalized Nash bargaining, there exist $\underline{\chi}$ and $\bar{\chi}$, with $\underline{\chi} < \bar{\chi}$ such that (1) if $\chi \geq \bar{\chi}$, the equilibrium is in the High-pledgeability Region; (2) if $\underline{\chi} \leq \chi < \bar{\chi}$, the equilibrium is in the Medium-pledgeability Region with $dW_B/d\chi|_{\bar{\chi}} < 0$; (3) if $\chi < \underline{\chi}$, the equilibrium is in the Low-pledgeability Region with $da/d\chi < 0$.*

We continue the numerical exercise using the functional forms and parameter values in Figure 3. Let $\theta = 0.5$. Figure 5 plots the equilibrium contracts with Nash Bargaining.

There are a few notable differences compared to the baseline model. For one thing, impatient depositors get lower consumption as pledgeability improves in Medium

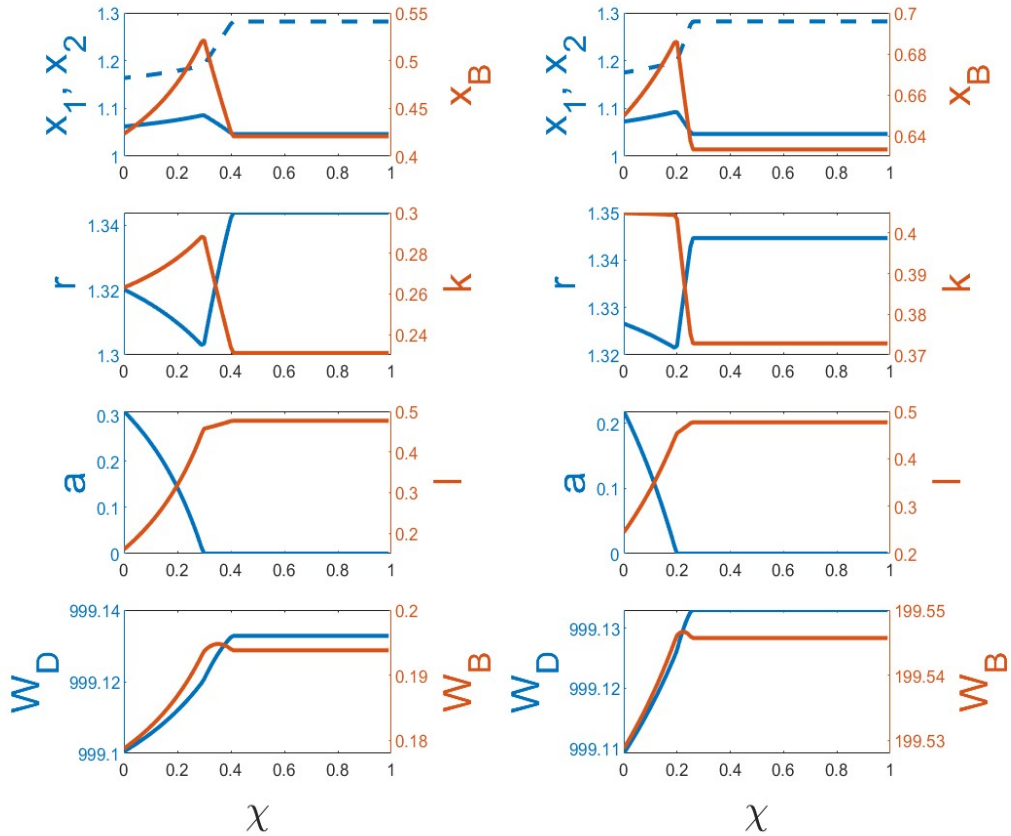


Figure 5: Nash Bargaining. Left: $\delta = 0.5$; Right: $\delta = 2$

Region, which is opposite to that in the competitive market. This is because risk sharing here depends on the pledgability.[17] Another notable difference is that loan size is positively related to pledgeability in the Low and Medium Regions, whereas it is nonmonotone in the baseline given that depositor's preference satisfies $CRRA > 1$.

5.2 Competitive Search

Suppose a bank can open a loan market that is characterized by the terms of borrowing $(x_1, x_2, x_B, r, k, \ell)$. Borrowers observe those posted terms and pay an entry cost, denoted by ϕ, choosing to go to a specific market. In a market, banks and borrowers are matched according to the matching function $M(n_D, n_B)$, where n_D and n_B are the measures of

[17] See equation (15)

banks and borrowers, respectively. Assume M is strictly increasing, strictly concave, and homogeneous of degree 1 in both arguments. Let $\tau = n_B/n_D$ be the market tightness. Let the probability that a bank meets a borrower be $\sigma(\tau) \equiv M(1,\tau)$ and the probability that a borrower meets a bank be $\sigma(\tau)/\tau$. Banks post terms of trade to solve the following problem:

$$\max_{x_1,x_2,x_B,r,k,a,\tau} \sigma(\tau)\left[\lambda u(x_1) + (1-\lambda) u(x_2)\right] + \left[1-\sigma(\tau)\right]\hat{W}_D \tag{19}$$

$$\text{st } (4), (5), (13), (14)$$

$$\frac{\sigma(\tau)}{\tau}\left\{-c(k) + v\left[(1-\lambda x_1 - a)\left(\bar{R}-r\right) + k\bar{R}\right]\right\} + \left[1-\frac{\sigma(\tau)}{\tau}\right]\hat{W}_B - \phi = \hat{W}_B \tag{20}$$

The LHS of (20) is the expected utility of a borrower if he chooses to enter the market, and the RHS is his payoff if he does not. Market entry occurs when the expected trade surplus is larger than ϕ.

The first-order conditions are given by (15), (16) and

$$\begin{aligned} \left[1-\varepsilon(\tau)\right] u'(x_1) S_B - \varepsilon(\tau) c'(k) S_D &= 0 && (21) \\ \eta - \frac{\sigma(\tau) u_2'(x_2)}{c'(k) - \chi v'(x_B)}\left[c'(k) - \bar{R} v'(x_B)\right] &= 0 && (22) \end{aligned}$$

where $\varepsilon(\tau) = \sigma'(\tau)\tau/\sigma(\tau)$ is the elasticity of the matching function and η is the Lagrangian multiplier associated with (14). With competitive search, bargaining power is endogenous: it is the elasticity of the matching function.[18]

As with the previous market structures, we find that the equilibrium falls into one of three regions associated with pledgeability as stated in the following proposition.

Proposition 3 *Consider the competitive search economy with $\varepsilon' < 0$. There exist $\underline{\chi}$ and $\bar{\chi}$, with $\underline{\chi} < \bar{\chi}$ such that (1) if $\chi \geq \bar{\chi}$, the equilibrium is in the High-pledgeability Region; (2) if $\underline{\chi} \leq \chi < \bar{\chi}$, the equilibrium is in the Medium-pledgeability Region; (3) if $\chi < \underline{\chi}$, the equilibrium is in the Low-pledgeability Region with $da/d\chi < 0$.*

[18]Note that if the matching elasticity is constant, the solution is the same as under Nash bargaining. In this special case, the market tightness is determined by (20).

The comparative statics under competitive search depend on the assumptions about the matching function and are generally ambiguous. There are a few exceptions. In the Low Region, with $\varepsilon' < 0$, we have $dk/d\chi > 0$ and $da/d\chi < 0$, and $dr/d\chi|_{\bar{\chi}} > 0$. The numerical analysis uses the same parameters as in Figure 3. Let $M(n_D, n_B) = n_D n_B/(n_D + n_B)$ and $\phi = 0.01$.

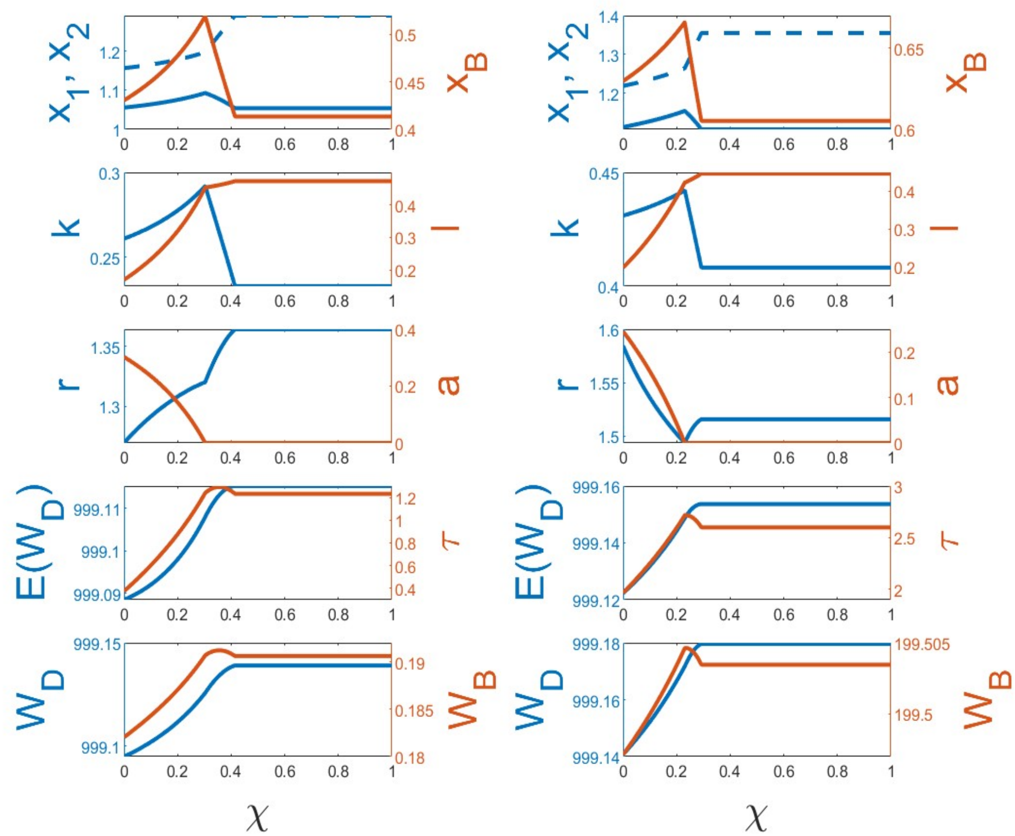


Figure 6: Competitive Search. Left: $\delta = 0.5$; Right: $\delta = 2$

Figure 6 plots the equilibrium contract terms. The contract variables are qualitatively similar to what we observed under Nash bargaining. One notable difference is the response by loan rates in the Low Region. In the other market structures, r either declines with χ or stays at $\underline{R}$. Here the loan rate may rise with pledgeability in the entire range (the left column). The market gets tighter when χ increases from 0, but it gets looser in the Medium Region and stays constant in the High Region. The ex-ante welfare of the borrowers is ϕ by the free entry condition, while the welfare of banks strictly increases for $\chi \leq \bar{\chi}$, despite the nonmonotone matching probability. For matched borrowers, welfare follows a pattern similar to that in Figure 5.[19]

6 Extensions

6.1 Lower Entry Cost for borrowers

We use a competitive search market structure to analyze the effects of lower entry costs for borrowers in the loan market. Intuitively, lower entry costs enable more borrowers to access the loan market, thereby increasing the number of borrowers served by banks. We study how changes in entry costs affect the terms of loan and deposit contracts.[20] Proposition 4 and the comparative statics are based on that $\varepsilon' < 0$. For $\varepsilon' > 0$, the results are ambiguous.

Proposition 4 *Suppose $\varepsilon' < 0$. The cutoffs, $\underline{\chi}$ and $\bar{\chi}$, strictly decrease in ϕ.*

The intuition behind Proposition 4 is as follows: a high entry cost discourages borrowers to enter the loan market, and the market becomes less tight. So banks provide more favorable terms to attract borrowers and depositors suffer by consuming less. Under the more favorable terms, borrowers are incentivized to repay their loans rather than default. Consequently, the repayment constraint is relaxed, the Hight-pledgeability Region expands, and the Low Region contracts.

Table 4 presents the comparative statics in each of the three regions. As ϕ increases, from the perspective of credit market indicators, credit conditions appear to improve,

[19]For the interested reader, we also consider a special case with take-it-leave-it offers in bilateral loan matches. The results of this special case are available from the authors upon request.

[20]This experiment can be done using other loan market structure. If the loan market is competitive, for example, we lose the High-pledgeability Region as no borrowers will enter given that ex post the profit is zero.

Table 4: Comparative Statics, Competitive Search, $\varepsilon' < 0$, Entry Cost

	$dx_1/d\phi$	$dx_2/d\phi$	$dx_B/d\phi$	$dk/d\phi$	$dr/d\phi$	$d\ell/d\phi$	$da/d\phi$
High	$-$	$-$	$+$	$-$	$-$	$+$	N.A.
Medium	$\pm$	$\pm$	$\pm$	$-$	$-$	$\pm$	N.A.
Low	$-$	$-$	$+$	$-$	$-$	$+$	$-$

reflected in lower interest rates and reduced collateral requirements. However, aggregate welfare declines as depositors are worse off by offering more favorable terms to borrowers.

We continue by extending the numerical example in Figure 6. Set $\chi = 0.3$ for the left column and $\chi = 0.23$ for the right. Then, let ϕ vary. For the left column, the cutoff between the Low and Medium Regions is $\phi = 0.012$. For $\phi > 0.047$, entering the loan market is not profitable given any feasible terms. For the right column, the cutoff between the Low and Medium Regions is $\phi = 0.010$, between the Medium and High Regions is $\phi = 0.092$. There is no entry for $\phi > 0.113$. Figure 7 plots the contract terms, which are consistent with our intuition and comparative statics. Moreover, with a higher ϕ, impatient depositors are worse off than patient depositors, implying that partial insurance is also weakened in this experiment.

In summary, lower entry costs encourage more borrowers to enter the market. However, the contract terms for each matched borrower become less favorable. Depositors, on the other hand, benefit from lower entry costs.

6.2 Financial Stability

Since there is no aggregate uncertainty in the model, a demand deposit with suspension clauses can strongly implement the "no run" equilibrium. To provide some insight into financial stability, we consider an unexpected negative productivity shock to the borrower's project. Depositors observe the decline at date $t = 1$. Denote the return by $\tilde{R}$, where $\tilde{R} < \bar{R}$. The borrower honors the repayment if

$$\ell \left(\tilde{R} - r\right) + k\tilde{R} \geq (1 - \chi)(\ell + k). \tag{23}$$

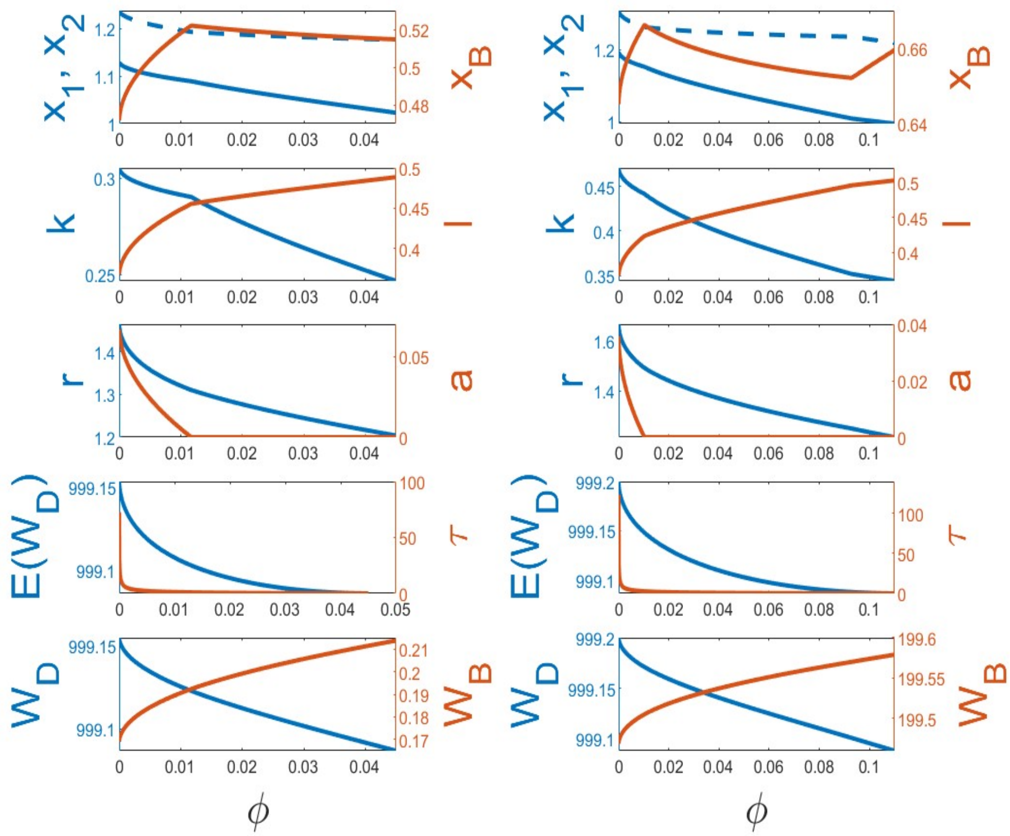


Figure 7: Optimal Contracts with different entry costs

Otherwise, he defaults.

Because the productivity shock is unexpected, contracts do not accommodate such an event. The bank may fail to fulfill its promise to pay patient depositors x_2. If $a > 0$, the bank can still make some payment at $t = 2$, but the payment may fall short of x_1. We assume that the depositors learn the borrower's repayment decision and then decide whether to withdraw at $t = 1$. Even with a suspension clause, knowing that there will be reduced resources available in $t = 2$ encourages early withdrawals.

In the High-pledgeability Region, (14) does not bind. As long as the fall in return is "small," borrowers honor the debt. However, in the Medium and Low Regions, the repayment constraint binds. A decrease in the borrower's return violates (14) and triggers a default. In the Medium Region, banks allocate all long-term investments to borrowers' projects. With all-the-eggs-in-one basket, the bank will have no resources to pay the depositors at $t = 2$. Thus, default by borrowers triggers a bank run. In the Low-pledgeability Region, the bank invests some resources in the safe project, which yields $a\underline{R}$. Consequently, the bank can pay each patient consumer $a\underline{R}/(1-\lambda)$ in $t = 2$. If $a\underline{R}/(1-\lambda) < x_1$, a bank run follows. However, if $a\underline{R}/(1-\lambda) \geq x_1$, it is incentive compatible that patient depositors do not withdraw at $t = 1$.

The implication is clear: with the extensive margin, banks are able to diversify their portfolios. In the presence of unanticipated shocks, the Medium-pledgeability Region and the lower portion of the High-pledgeability Region are more fragile than the Low-pledgeability Region. When χ falls below a critical threshold, the Low-pledgeability Region implements a diversified portfolio that reduces the chance of a bank run.

We extend the numerical analysis in Figure 6 with $\lambda = 0.2$, $\gamma = 1.1$, and $A = 0.05$. The cutoffs are $\underline{\chi} = 0.44$ and $\bar{\chi} = 0.71$. For $\chi \geq 0.71$, a higher pledgeability allows the system to absorb larger negative shocks. In the Low-pledgeability Region, for $\chi < 0.12$, condition $a\underline{R} > (1-\lambda)\,x_1$ is satisfied, indicating that even if the borrowers default, bank failure does not occur.

6.3 Separated vs Unified Banking

The Glass-Steagall Act of 1933 distinguishes between loans and direct investments by restricting commercial banks from participating in securities underwriting.[21] Our goal in this section is to assess the impact of the Act on depositor's welfare. To adapt our model for this task, we assume that the return on bank's direct investment technology

[21]Kroszner and Rajan (1994) examine the performance of commercial banks, comparing those constrained from direct investment with investment banks. Investment banks are free to invest directly. They present evidence consistent with the notion that there is no substantial difference between the quality of securities underwritten by investment banks and loans made by commercial banks. Gorton and Schmid (2000) study the German banking environment. They argue that unified banking results in better returns by firms receiving direct investment from banks. Berlin and Meister (1999) examine the potential contributions of bank equity investments in addressing the challenges faced by financially distressed businesses. Additionally, Santos (1998) delved into the welfare implications of restrictions on banks' direct investments in non-financial enterprises.

is stochastic. It yields $\underline{R}$, where $1 < \underline{R} < \bar{R}$, with probability ρ, and 0 with probability $1 - \rho$. Yields are bank-specific, and depositors cannot diversify across banks due to, for example, geographic or legal restrictions. Without the Act, banks can choose a portfolio that includes storage, risky direct investments, and loans to external borrowers. We refer to this as a "unified banking system." Under the Act, banks are limited to storage or loans, creating a "separated banking system." Thus, we interpret the direct investment asset as the key distinguishing feature of the unified banking system.

Without further friction, even if there is risk in production and thus uncertainty in the deposit rates, the unified system dominates the separated system, as it provides an additional option for the bank. Here, we introduce a deposit insurance program that guarantees the same return for all banks in all states. Banks pay a premium ψ at $t = 0$. This insurance program aggregates idiosyncratic risks across banks and provides subsidies to those experiencing adverse shocks from direct investments.[22] However, such an insurance program may cause excessive investment in the risky technology.

A deposit contract under separated banking solves the following:

$$
\begin{aligned}
W_D^S = \max_{x_1, x_2, s} \lambda u(x_1) + (1-\lambda) u(x_2) \\
\text{st. } (1-\lambda) x_2 = (1 - \lambda x_1 - s) r + s \\
x_2 \geq x_1
\end{aligned}
$$

where s is the investment in storage. The first-order condition wrt x_1 is (6). The choice of s is $s = 0$ if $r > 1$, and $s > 0$ if $r = 1$. The equilibrium r can vary between 1 and $\bar{R}$. The borrower's problem remains the same as in Section 4. The market clearing condition is $1 - \lambda x_1 - s = n_B \ell$. The equilibrium under the separated banking can be viewed as a special case of Section 4 with $\underline{R} = 1$.

[22]The deposit insurance policy is exogenously set by the government. Deposit insurance is not the solution to a Ramsey problem. See Davila and Goldstein (2023) for a modified version of Diamond-Dybvig in which the optimal deposit insurance is derived.

Under the unified banking system, the deposit contract solves:

$$W_D^U(\psi) = \max_{x_1, x_2, a} \lambda u(x_1) + (1-\lambda) u(x_2)$$

$$\text{st. } (1-\lambda) x_2 = (1 - \psi - \lambda x_1 - a) r + a\underline{R} \tag{24}$$

$$x_2 \geq x_1 \tag{25}$$

Depositors are shielded from uncertainty by the insurance program, so they treat the return on direct investment as $\underline{R}$ with probability 1. The first order condition with respect to x_1 is (6). If $r > \underline{R}$, then $a = 0$. If $r = \underline{R}$, then $a \in (0, 1 - \psi - \lambda x_1)$. If $r < \underline{R}$, then $a = 1 - \psi - \lambda x_1$. The borrower's problem remains the same as in Section 4. The market clearing condition is (11). Because $1 - \rho$ is the fraction of banks realizing no return to the direct investment, an actuarially fair insurance premium is $\psi = a(1-\rho)\underline{R}$. This closes the model.

Let $\underline{\chi}^U$ be the cutoff χ between the Low and Medium Regions defined in Section 4 with $\psi = 0$. By Proposition 1, if $\chi \geq \underline{\chi}^U$, then $r \geq \underline{R}$ and $a = 0$. Banks do not invest in direct investment and the outcome is the same in both systems. This implies that when the loan market friction is minimal, the Act has no effect. In the following, we will focus on the case where $\chi < \underline{\chi}^U$.

It is straightforward to show that $\partial W_D^U / \partial \rho > 0$. As direct investment becomes less risky, the bank's portfolio options improve. The depositor's welfare under the Act, W_D^S, does not change with ρ. Therefore, there exists a cutoff ρ above which the Act harms the economy and below which it does not.

Given ρ, what if χ changes in the interval of $\left[0, \underline{\chi}^U\right]$? First, consider the case of separated banking. Suppose χ decreases in $\left[0, \underline{\chi}^U\right]$. According to the comparative statics in Table 3, depositor's welfare decreases in the Low-pledgeability Region. Next, consider unified banking. For $\chi < \underline{\chi}^U$, banks invest in risky technology, and the return is constant at $\underline{R}$. Since some banks fail in direct investment, the insurance premium is positive. By paying these premiums, resources are diverted from higher-return production and excessive investment is made in risky technology. Therefore, as χ decreases from $\underline{\chi}^U$, the welfare also decreases. It is not straightforward to determine whether the Act yields higher welfare.

We use a numerical example to illustrate the result. Continue with the parameters in Figure 3 with $\alpha = 1$, $\underline{R} = 1.1$, $\lambda = 0.3$, and $\delta = 2$. Figure 8 plots the frontier in the (χ, ρ) space where the depositor's welfare is the same under both systems. The dark-shaded area represents where unified banking dominates separated banking, while the light-shaded area indicates where separated banking outperforms unified banking. The boundary is nonmonotonic in χ. For a given χ, the unified system yields higher depositor welfare if ρ is high, and vice versa if ρ is low. For a given ρ, it is not clear which system is better when χ changes. In the example, there are multiple cutoffs of χ for medium values of ρ. Separated banking dominates unified banking for both low and high values of χ. However, for medium values of χ, separated banking can actually reduce depositor's welfare. Thus, our results suggest that the impact of the Act depends not only on the riskiness of direct investment but also on the friction in the alternative asset market. Thus, there is no single measure to definitively determine whether the Act improves welfare.

7 Conclusion

In this paper, we examine the interaction between asset pledgeability and banks' optimal loan and deposit contracts in a general equilibrium framework. Our theory reveals that contract terms exhibit intricate interactive patterns as pledgeability changes. Such interactions lead to predictions that directly contradict the conventional wisdom derived from partial-equilibrium analysis. These theoretical results align with empirical evidence on the impact of BAPCPA on the loan market. Moreover, the results bear on measures of credit conditions that rely on monotonic relationships between loan contract terms and changes in pledgeability.

Market structures play a crucial role in determining how lenders and borrowers

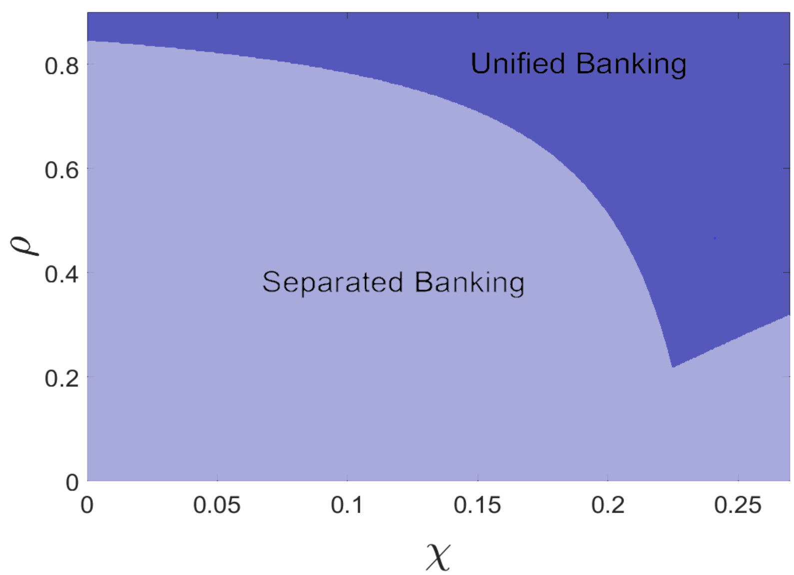


Figure 8: Universal Banking vs Narrow Banking

divide the gains from trade as the repayment constraint relaxes with improved pledgeability. Although the underlying economic mechanisms differ, the contract terms are nonmonotonic in pledgeability across all common market structures we study.

We extend the model to consider important banking issues such as changes in entry costs, financial fragility, and the impact of the Glass-Steagall Act. In each subject, our model provides new insight: with lower entry costs, induces borrowers to compete for funds, and depositors actually benefit; economies with low pledgeability can be more stable because banks diversify their portfolio; and whether the Glass-Steagal Act improves depositor's welfare depends on investment riskiness as well as the friction in other markets.

Our future work includes extending the model to incorporate dynamics to better understand the differences in trends before and after BAPCPA, as well as examine stochastic pledgeability at growth or business-cycle frequencies. Furthermore, we aim to explore how systemic changes in credit conditions affect heterogeneous agents and to revisit the data to further validate our theoretical framework.

Appendix

A Figures

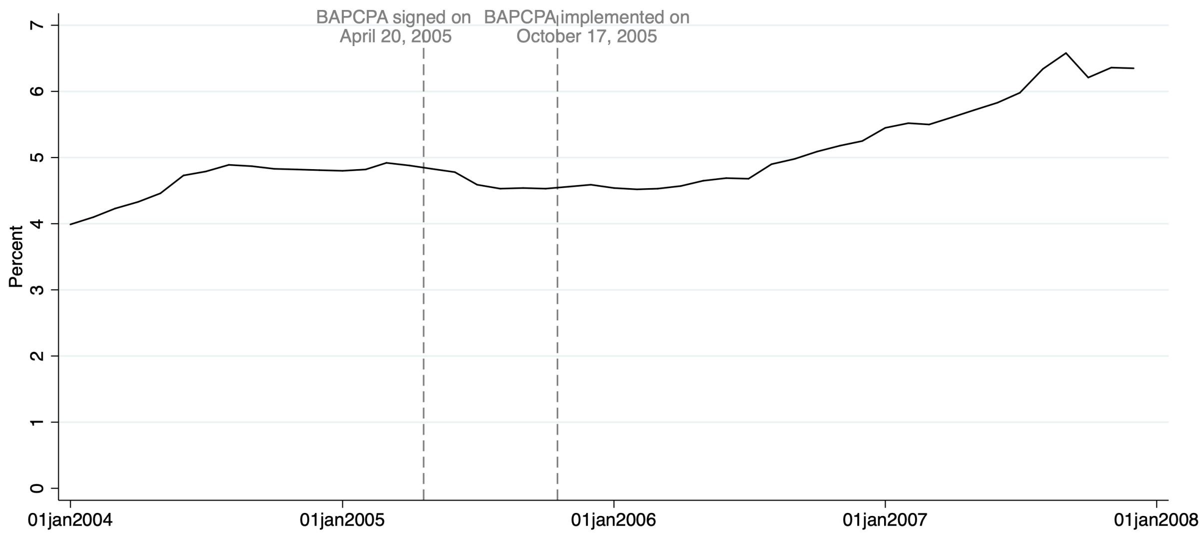
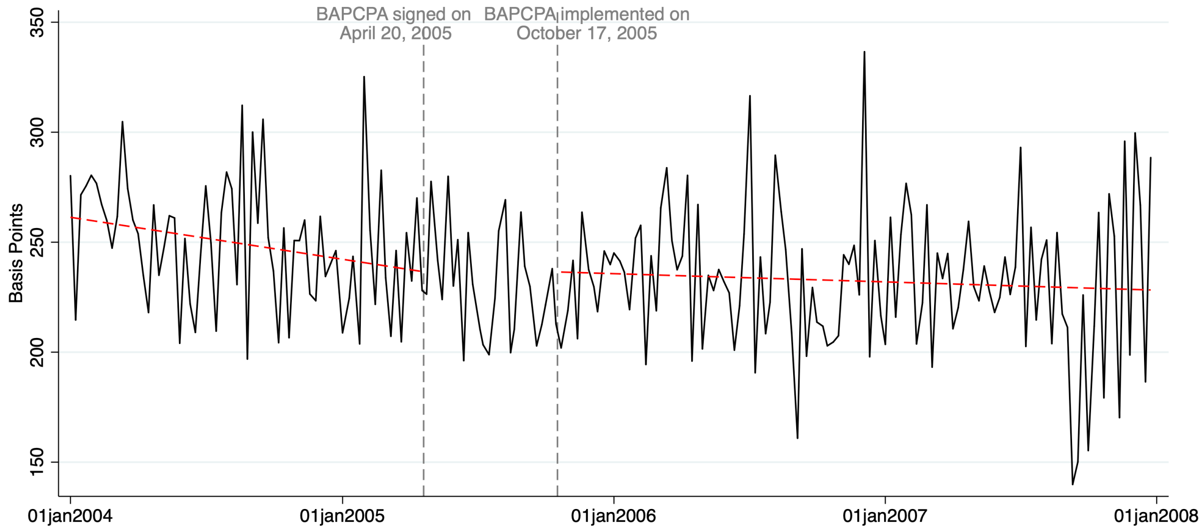
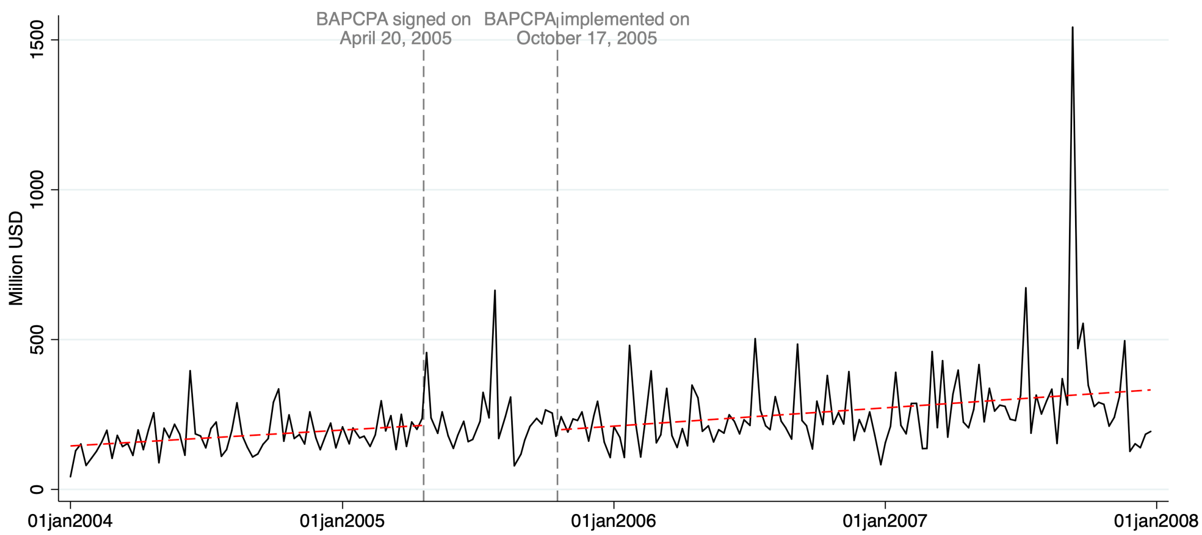


Figure A.1: 3-month London Interbank Offered Rate (LIBOR)

Note: This figure displays the monthly time series of the 3-month London Interbank Offered Rate (LIBOR) in the United Kingdom from 2004 to 2007, measured in percent. The two vertical dashed gray lines mark the passage of BAPCPA on April 20, 2005, and its implementation on October 17, 2005.



Panel A. Average Spreads of Loans



Panel B. Average Sizes of Loans

Figure A.2: Weekly Average Spreads and Sizes of Loans

Note: This figure shows the average spreads of loans (Panel A) and the average loan sizes (Panel B) in Dealscan from 2004 to 2007. We aggregate the daily averages into a weekly frequency based on the active dates of the loans. The two vertical dashed gray lines mark the passage of BAPCPA on April 20, 2005, and its implementation on October 17, 2005. In each panel, we plot two red fitted lines: one representing the period before BAPCPA was passed and the other for the period after its implementation. These fitted lines are linear predictions of the variable of interest over time.

B Proofs

Proof of Proposition 1: It is straight-forward to show that if $\chi \geq \bar{\chi}$, the equilibrium is in the High-pledgeability Region, which entails $\eta = 0$. Now consider $\chi < \bar{\chi}$. Then $\eta > 0$. Consider the Low-pledgeability Region which requires $a > 0$ and $r = \underline{R}$. By (6), $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$. By (7)-(9), the demand for ℓ is characterized by

$$-c'\left(\frac{1-\chi+\underline{R}-\bar{R}}{\bar{R}-1+\chi}\ell\right)\left(1-\chi+\underline{R}-\bar{R}\right)+(1-\chi)\,\underline{R}v'\left(\frac{1-\chi}{\bar{R}-1+\chi}\ell\right)=0 \tag{26}$$

Take derivative wrt χ to get

$$\frac{d\ell}{d\chi}=-\frac{c'\left(\bar{R}-\underline{R}\right)\left(\bar{R}-1+\chi\right)^2+\ell\underline{R}\,(1-\chi)\left[c''\left(1-\chi+\underline{R}-\bar{R}\right)-v''\underline{R}\bar{R}\right]}{(1-\chi)\left(\bar{R}-1+\chi\right)\left[\underline{R}^2(1-\chi)^2v''-\left(1-\chi+\underline{R}-\bar{R}\right)^2c''\right]}>0$$

By (11) and since x_1 is constant, $da/d\chi < 0$. Thus, the cutoff of $\underline{\chi}$ between the Medium- and Low-pledgeability Regions is unique. At $\underline{\chi}$, ℓ solving (26) satisfies $\ell = 1 - \lambda\hat{x}_1$. ■

Proof of Proposition 2: The system of equations for comparative statics in Low-pledgeability Region is (13)-(17) with $x_B = (1-\chi)(1-\lambda x_1 - a + k)$ and $r = \underline{R}$. Totally differentiate wrt x_1, k, a, and χ. Let $u_t' = u'(x_t)$ and $u_t'' = u''(x_t)$. After some algebra, we get

$$
\begin{aligned}
\frac{da}{d\chi} \doteq\ & u_1'' \left[\left(1-\chi+\underline{R}-\bar{R}\right) c'' - \underline{R}\left(1-\chi\right)^2 v'' \right] \left[\theta u_1' v' + \left(1-\theta\right) c' u_2' \right] \left(1-\lambda x_1 - a + k\right) \\
& + \frac{\underline{R}\lambda\left(\bar{R}-1+\chi\right)}{1-\lambda} u_2'' \left(1-\chi+\underline{R}-\bar{R}\right) c'' \left[\theta u_1' v' + \left(1-\theta\right) c' u_2' \right] \left(1-\lambda x_1 - a + k\right) \\
& + \frac{\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda} u_2'' \left[-c' + \underline{R} v' + \underline{R}\left(1-\chi\right)\left(1-\lambda x_1 - a + k\right) v'' \right] \times \\
& \left[\theta\left(u_1'' S_B - \left(1-\chi\right)\lambda u_1' v'\right) - \left(1-\theta\right)\lambda\left(1-\chi+\underline{R}-\bar{R}\right) u_2' c' \right] \\
& + \frac{\lambda \underline{R}^2 \left(1-\chi\right)^2}{1-\lambda} \left(1-\lambda x_1 - a + k\right) v'' u_2'' \times \\
& \left\{ \theta u_1' \left[-c' + \left(1-\chi\right) v' \right] - \left(1-\theta\right) \left[c'' S_D + \left(\bar{R}-\chi\right) c' u_2' \right] \right\} \\
& - \frac{\underline{R}}{1-\lambda} \left(1-\lambda x_1 - a + k\right) u_2'' \left[\left(1-\chi+\underline{R}-\bar{R}\right) c'' - \underline{R}\left(1-\chi\right)^2 v'' \right] \\
& \times \left[\theta\left(u_1'' S_B - \left(1-\chi\right)\lambda u_1' v'\right) - \left(1-\theta\right)\lambda\left(1-\chi+\underline{R}-\bar{R}\right) u_2' c' \right] \\
& + \left[u_1'' + \frac{\underline{R}\lambda\left(\bar{R}-1+\chi\right)}{1-\lambda} u_2'' \right] \left[-c' + \underline{R} v' + \underline{R}\left(1-\chi\right)\left(1-\lambda x_1 - a + k\right) v'' \right] \\
& \times \left\{ \theta u_1' \left[-c' + \left(1-\chi\right) v' \right] - \left(1-\theta\right) \left[c'' S_D + \left(\bar{R}-1+\chi\right) c' u_2' \right] \right\}
\end{aligned}
$$

which is negative.

Proof of Proposition 3: The system of equations for the Low-pledgeability Region is (5), (13) (14), (15), (16), (20), and (21) with $x_B = \left(1-\chi\right)\left(1-\lambda x_1 - a + k\right)$. We can substitute x_2, x_B and r by the linearity of the constraints and then jointly solve (x_1, k, a, τ). By the implicit function theorem, $da/d\chi = |A_2|/|A_1|$, where $|A_1|$ =

$$
\begin{aligned}
& \sigma \frac{\varepsilon'}{\varepsilon} u_1' S_B \times \\
& \begin{vmatrix}
c' \frac{\bar{R}-1+\chi}{\underline{R}} & \left(1-\chi\right)\lambda v' & \left(1-\chi\right) v' \\
c''\left(1-\chi+\underline{R}-\bar{R}\right) + \underline{R}\left(1-\chi\right)^2 v'' & \underline{R}\left(1-\chi\right)^2 \lambda v'' & \underline{R}\left(1-\chi\right)^2 v'' \\
-\frac{\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda} u_2'' & u_1'' + \frac{\lambda\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda} u_2'' & -\frac{\underline{R}\left(1-\chi+\underline{R}-\bar{R}\right)}{1-\lambda} u_2''
\end{vmatrix} \\
& - \left(1-\varepsilon\right)\phi \times \\
& \begin{vmatrix}
u_2' c' \left(\bar{R}-1+\chi\right) + \varepsilon c'' S_D & c' u_2' \lambda \left(1-\chi+\underline{R}-\bar{R}\right) - \left(1-\varepsilon\right) u_1'' S_B & u_2' c' \left(\chi+\underline{R}-\bar{R}\right) \\
c''\left(1-\chi+\underline{R}-\bar{R}\right) + \underline{R}\chi^2 v'' & \underline{R}\left(1-\chi\right)^2 \lambda v'' & \underline{R}\left(1-\chi\right)^2 v'' \\
-\frac{\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda} u_2'' & u_1'' + \frac{\lambda\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda} u_2'' & -\frac{\underline{R}\left(1-\chi+\underline{R}-\bar{R}\right)}{1-\lambda} u_2''
\end{vmatrix}
\end{aligned}
$$

is positive if $\varepsilon' < 0$, and $|A_2| =$

$$
\begin{aligned}
&-\sigma\frac{\varepsilon'}{\varepsilon}u_1' S_B \times \\
&\begin{vmatrix}
\frac{\bar{R}-1+\chi}{\underline{R}}c' & (1-\chi)\lambda v' & (1-\lambda x_1 - a + k)\, v' \\
\left(1-\chi+\underline{R}-\bar{R}\right)c'' + \underline{R}(1-\chi)^2 v'' & \underline{R}(1-\chi)^2\lambda v'' & \begin{array}{l} -\frac{\bar{R}-\underline{R}}{1-\chi}c' \\ +\underline{R}(1-\chi)^2(1-\lambda x_1 - a + k)\, v'' \end{array} \\
-\frac{\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda}u_2'' & u_1'' + \frac{\lambda\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda}u_2'' & -\frac{\underline{R}}{1-\lambda}(1-\lambda x_1 - a + k)\, u_2''
\end{vmatrix} \\
&+(1-\varepsilon)\phi \times \\
&\begin{vmatrix}
u_2' c'\left(\bar{R}-1+\chi\right) + \varepsilon c'' S_D & \begin{array}{l} c' u_2' \lambda\left(1-\chi+\underline{R}-\bar{R}\right) \\ -(1-\varepsilon)\, u_1'' S_B \end{array} & \begin{array}{l} \left[\varepsilon c' u_2' + (1-\varepsilon)\, u_1' v'\right] \times \\ (1-\lambda x_1 - a + k) \end{array} \\
\left(1-\chi+\underline{R}-\bar{R}\right)c'' + \underline{R}(1-\chi)^2 v'' & \underline{R}(1-\chi)^2\lambda v'' & \begin{array}{l} \underline{R}(1-\chi)^2 v'' \times \\ (1-\lambda x_1 - a + k) - \frac{\bar{R}-\underline{R}}{1-\chi}c' \end{array} \\
-\frac{\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda}u_2'' & u_1'' + \frac{\lambda\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda}u_2'' & -\frac{\underline{R}\left(1-\chi+\underline{R}-\bar{R}\right)}{1-\lambda}u_2''
\end{vmatrix}
\end{aligned}
$$

is negative if $\varepsilon' < 0$. So $da/d\chi < 0$, and there is a unique cutoff, denoted by $\underline{\chi}$, between the Medium- and Low-pledgeability Regions. ■

Proof of Proposition 4: In the High-pledgeability Region, the equations contract are (5), (7), (20), (21) and $c'(k) = \bar{R}u'(x_2)$, which jointly solve (x_1, k, r, τ). By the implicit function theorem, given $\varepsilon' < 0$, $d\bar{\chi}/d\phi = |A_3|/|A_1|$, where $|A_1|$ is defined in the proof of prop 4 and

$$
|A_3| = \frac{\varepsilon'}{\varepsilon}\tau u_1' S_B \begin{vmatrix}
c''\left(1-\chi+\underline{R}-\bar{R}\right) - \underline{R}(1-\chi)^2 v'' & \underline{R}(1-\chi)^2\lambda v'' \\
\frac{\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda}u_2'' & -u_1'' + \frac{\lambda\underline{R}\left(\bar{R}-1+\chi\right)}{1-\lambda}u_2''
\end{vmatrix}
$$

Given $\varepsilon' < 0$, $|A_3| < 0$, and $da/d\phi < 0$, which means given χ, if $\phi' > \phi$ and $a > 0$ under ϕ', then $a > 0$ under ϕ. Thus, the cutoff of $\underline{\chi}$ is lower for higher ϕ. ■

References

[1] F. Allen & D. M. Gale (2003) "Financial Fragility, Liquidity and Asset Prices," *Journal of the European Economic Association* 2(6), 1015–48.

[2] M. Amador & J. Bianchi (2024) "Bank Runs, Fragility, and Credit Easing, " *American Economic Review* 114(7), 2073-2110.

[3] G. Antinolfi & S. Prasad (2008) "Commitment, Banks and Markets," *Journal of Monetary Economics* 55(2), 265-277.

[4] A. N. Berger & G. F. Udell (1990) "Collateral, Loan Quality and Bank Risk," *Journal of Monetary Economics* 25(1), 21-42.

[5] A. N. Berger & G. F. Udell (1995) "Relationship Lending and Lines of Credit in Small Firm Finance," *Journal of Business* 351-381.

[6] M. Berlin & L. J. Meister (1999) "Deposits and Relationship Lending," *The Review of Financial Studies* 12(3), 579-603.

[7] B. Bernanke & M. Gertler (1990) "Financial Fragility and Economic Performance," *Quarterly Journal of Economics* 105(1), 87-114.

[8] E. Benmelech & N. K. Bergman (2009) "Collateral Pricing, " *Journal of Financial Economics* 91(3), 339-360.

[9] D. Besanko & A. V. Thakor (1987) "Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets," *International Economic Review* 671-689.

[10] S. Bhattacharya & D. Gale (1987) "Preference Shocks, Liquidity and Central Bank Policy," *New Approaches to Monetary Economics* 69-88.

[11] B. Biais, T. Mariotti, G. Plantin & J. C. Rochet (2007) "Dynamic Security Design: Convergence to Continuous Time and Asset Pricing Implications," *Review of Economic Studies* 74(2), 345-390.

[12] A. W. A. Boot, A. Thakor & G.F. Udell (1991) "Secured Lending and Default Risk: Equilibrium Analysis, Policy Implications and Empirical Results," *Economic Journal* 101(406), 458-72.

[13] S. A. Brave & D. L. Kelly (2017) "Introducing the Chicago Fed's New Adjusted National Financial Conditions Index," *Chicago Fed Letter*, Federal Reserve Bank of Chicago.

[14] J. Bryant (1980) "A Model of Reserves, Bank Runs, and Deposit Insurance, " *Journal of Banking and Finance* 4(4), 335-344.

[15] R. J. Caballero & A. Krishnamurthy (2004) "Fiscal Policy and Financial Depth," *National Bureau of Economic Research* 10532.

[16] S. Chava & M. R. Roberts (2008) "How Does Financing Impact Investment? The Role of Debt Covenants," *The Journal of Finance* 63(5), 2085–2121.

[17] E. Davila & I. Goldstein (2023) "Optimal Deposit Insurance," *Journal of Political Economy* 131(7), July, 1676-1730.

[18] P. M. DeMarzo & M. J. Fishman (2007) "Optimal Long-term Financial Contracting," *Review of Financial Studies* 20(6), 2079-2128.

[19] D. W. Diamond & P. H. Dybvig (1983) "Bank Runs, Deposit Insurance, and Liquidity, " *Journal of Political Economy* 91(3), 401-419.

[20] D. W. Diamond (1997), "Liquidity, Banks, and Markets," *Journal of Political Economy* 105(5), 928-956.

[21] K. Dowd (1992) "Optimal Financial Contracts," *Oxford Economics Papers* 44(3), October, 672-93.

[22] E. Farhi, M. Golosov & A. Tsyvinski (2009), "A Theory of Liquidity and Regulation of Financial Intermediation," *Review of Economic Studies* 76(3), 973-992.

[23] G. Gorton & F. A. Schmid (2000), "Universal Banking and the Performance of German Firms," *Journal of Financial Economics* 58, 29-80.

[24] T. Gross, R. Kluender, F. Liu, M. Notowidigdo & J. Wang (2020) "The Economic Consequences of Bankruptcy Reform," *American Economic Review* 111(7), 2309-41.

[25] A. Gupta, H. Sapriza & V. Yankov (2022) "Yhje Collateral Channel and Bank Credit," *Finance and Economics Discussion Series* 2022-024, Washington, D.C.: Federal Reserve Board.

[26] M. Hellwig (1994) "Liquidity Provision, Banking, and the Allocation of Interest Rate Risk," *European Economic Review* 38(7), 1363-1389.

[27] D. Hester (1979) "Customer relationships and terms of loans," *Journal of Money, Credit, and Banking* 11, 349-57.

[28] B. Holmström & J. Tirole (1998) "Private and Public Supply of Liquidity," *Journal of Political Economy* 106(1) 1-40.

[29] C. J. Jacklin (1987) "Demand Deposits, Trading Restrictions, and Risk Sharing," *Contractual Arrangements for Intertemporal Trade* 26-47.

[30] U. Jerman & V. Quadrini (2012) "Macroeconomic Effects of Financial Shocks,"*Economic Review* 102, 238-71.

[31] K. John, A. W. Lynch & M. Puri (2003) "Credit Ratings, Collateral, and Loan Characteristics: Implications for Yield," *Journal of Business* 76(3), 371-409.

[32] E. Kalai (1977) "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," *Econometrica* 45, 1623-30.

[33] N. Kiyotaki & J. Moore (1977) "Credit Cycles," *Journal of Political Economy* 105, 211-48.

[34] L. Klapper (1999) "The Uniqueness of Short-term Collateralization," *Policy Research Working Paper Series 2544* The World Bank.

[35] R. Kroszner & R. G. Rajan (1994) "Is the Glass-Steagall Act Justified: A Study of the U.S. Experience with Universal Banking before 1933," *American Economic Review* 84(4), 810-32.

[36] R. Posner (2005) "The Bankruptcy Reform Act," *Becker-Posner Blog* "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons."

[37] G. Rocheteau, R. Wright & C. Zhang (2018) "Corporate Finance and Monetary Policy," *American Economic Review* 108(4-5), 1147-86.

[38] J. Santos (1998) "Commercial Banks in the Securities Business: A Review," *Federal Reserve Bank of Cleveland Working Paper 9610*.

[39] J. Sprayregen, R. Cieri, & R. Wynne (2005) "Alert: Bankruptcy Abuse and Consumer Protection Act of 2005," *Kirkland & Ellis LLP*.

[40] E. L. Von Thadden (1999) "Liquidity Creation through Banks and Markets: Multiple Insurance and Limited Market Access," *European Economic Review* 43, 991-1006.

[41] S. D. Williamson (1987) "Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing," *Journal of the European Economic* 102(1), 135-145.